

**Parabola Solar Heater** can get strong heat with less heat up time, but expensive, sensitive to the solar direction, and rather dangerous for naked eye (→ sun glass).

[http://www.solarcooker-at-cantinawest.com/parabolic\\_solar\\_cooker\\_solar\\_burner.html](http://www.solarcooker-at-cantinawest.com/parabolic_solar_cooker_solar_burner.html)

**Box type heater with a reflector plate** is easy, but heat up time is longer.

[http://solarcooking.wikia.com/wiki/Minimum\\_Solar\\_Box\\_Cooker](http://solarcooking.wikia.com/wiki/Minimum_Solar_Box_Cooker)

Author tried to gain merits of both methods, which is **4 reflector plate with inner heat shield box** (above fig). It can scarcely get **over 100°C** for **3kg water in a kettle** in **fine winter day** in **about 2~3 hours**. **Rotational setting** is effective to get max T, but rather inconvenient.

**Input heat** is proportional to **input mouth area** ( $0.9 \text{ m} \times 0.9 \text{ m}$ ), while **loss heat output** is rather complicated due to **surface temperature distribution** of outer shield box.

The loss heat is mainly due to **random wind ventilation of the box surface** (highest temperature distribution in especially top input glass surface and the frame portion).

**Bottom and side wall** temperature may be lower due to double box shield method.

**Water boiling is most expensive**, so the cost is reduced, but not nothing in cloudy days

Aim at here is not recommending solar set at here, but **theoretical heat equations** which are useful to **estimate** the performance {max T, heat up time, volume of load}.

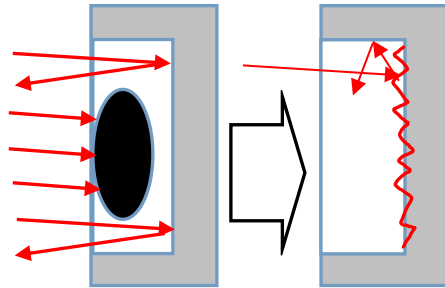
**[1] : Heat Account.**

(1) **Max Solar Heat input**  $\equiv J_S = 0.9m \times 0.9m \times (1000W/m^2; 1360W/m^2) = 800W$ .

**Total Heat Input Mouth Area:**  $S_S = 0.9m \times 0.9m$  . <<Inner Box Mouth =  $0.45m \times 0.45m$ >>

\* This is decisive element determining **max T** and heat up time =  $\tau$  .Of course, larger is better, while larger increases damage risk by strong wind .

\* \* **direct return loss** by reflection from inner heat box wall.



The flat reflection wall is not good, so **back wall** should be random face for making **random reflection** not to go out <random winding lines>. **Black wall** is best to absorb, however **higher wall temperature** causes more heat loss.

(2) **Max Heat Dissipation output**  $\equiv J_D = J_R + J_V + J_{cd} = 800W$  (at max temperature  $\sim 105^\circ C$ ).

\* Inner Box Surface Area:  $S_{inner} = 2 \times 0.45 \times 0.45 + 4 \times 0.45 \times 0.2 = 0.77m^2$ .

Larger box volume increases heat loss and no utilizable heat capacity.

\* Outer Box Surface Area:  $S_{outer} = 2 \times 0.6 \times 0.6 + 4 \times 0.6 \times 0.3 = 1.44m^2$ .

Note heat loss is entirely due to outer box surface temperature =  $T(S_{outer})$ . Especially top surface =  $S_{bt}$  (top glass and top surface frame) are not serious.

(a) **surface air ventilation loss**: {  $\kappa_v = 7W/m^2^\circ C$  in room, but larger in wind field }

$J_{vbt} = S_{bt} \kappa_v (T_{bt} - T_E) = (0.6m \times 0.6m) \times 2.0 \times 7W/m^2^\circ C \times 80^\circ C \sim 400W$ .....top surface

$J_{vbs} = S_{bs} \kappa_v (T_{bs} - T_E) = (0.6m \times 0.6m + 4 \times 0.6 \times 0.3) \times 2.0 \times 7W/m^2^\circ C \times 20^\circ C \sim 300W$ ..side wall

☞ : The value "2.0" is author's coarse estimation for weak wind velocity < 10m/s.

☞ : In strong windy days, temperature can not reach  $100^\circ C$ , but such as about  $70^\circ C$ .

(b) **AI surface radiation loss**: {  $\alpha = 0.05$  }  $J_R = S \alpha \sigma (T_B^4 - T_E^4)$ .  $\sigma = 5.67 \times 10^{-8} J/m^2K^4$

$J_{Rbt} = (0.6m \times 0.6m) \times 0.05 \times 5.67 \times 10^{-8} < (273+110)^4 - (273+10)^4 > = 15W$ .

$J_{Rbs} = (0.6m \times 0.6m + 4 \times 0.6 \times 0.3) \times 0.05 \times 5.67 \times 10^{-8} < (273+30)^4 - (273+10)^4 > = 6W$ .

☞ : Thus **surface radiation loss** is rather weak by {  $\alpha = 0.05$  for AI } . Neglecting radiation loss would not exceed over **10% error** in about  $100^\circ C$ . This is **approximation** in the report.

(c) **Heat leakage air flow from top glass frame is not counted in above calculation.**

Note top glass can **not completely shield inner air**, but some degree is leaking at anytime.

This heat amount may be serious.  $\sim 100W?$

(d) **Thickness of Top Input Glass** <5mm→8mm>.

This is the **highest temperature surface**, which is expected less heat output.

**Top glass thickness is dominant to get higher max temperature.**

$$J_{vbt} = S_{bt} \kappa_v (T_{bt} - T_E) = (0.6m \times 0.6m) \times 2.0 \times 7 W/m^2 \cdot ^\circ C \times 80^\circ C \sim 400W \dots \text{top surface}$$

$$J_{cbt} = S_{bt} \kappa_c (T_{bi} - T_{bt}) / d = 400W \quad \ll \text{heat transfer from in to out by conduction} \gg$$

$$\{ \kappa_c = 0.55 \sim 0.75 W/mK; \quad S_{bt} = 0.45m \times 0.45m; \quad d = 0.005m \}$$

$$(T_{bi} - T_{bt}) = d \times J_{cbt} / S_{bt} \kappa_c = 14 \sim 19^\circ C \quad (d = 0.005m),$$

$$G \equiv (d + \Delta d) / d, \rightarrow G = 1.6, \quad (T_{bi} - T_{bt}) = 22 \sim 30^\circ C$$

(e) **Heat transfer from inner box to outer one is air ventilation at inner heat box.**

$$* \text{ Inner Box side Surface Area: } S_{inner-s} = 0.45 \times 0.45 + 4 \times 0.45 \times 0.2 = 0.57m^2.$$

$$* \text{ Outer Box side Surface Area: } S_{outer-s} = 0.6 \times 0.6 + 4 \times 0.6 \times 0.3 = 1.1m^2.$$

$$J_{cibs} = S_{inner-s} \kappa_v (T_{bi} - T_{bt}) = 0.57m^2 \times 7 \times (100 - 30) = 280W.$$

$$J_{cibs} = S_{inner-s} \kappa_v (T_{bi} - T_{bt}) = 0.57m^2 \times 7 \times (110 - 30) = 300W.$$

$$J_{cbt} = S_{bt} \kappa_c (T_{bi} - T_{bt}) / d = (0.57 + 1.1) / 2 \times 0.026 \times (110 - 30) / 0.1m = 17W.$$

\* **conduction heat transfer** is very weaker than that of ventilation.

(f) **weaken air ventilation between inner and outer box space.**

Inner box should be shielded to **reduce air venation**, which could be accomplished by inserting bubble polystyrene, or something lower conductivity. Then note **heat box temperature exceeds over 100°C**, so those must be tough for heating.

(g) Following are **summary table** of this report.

<b>Max Heat Input</b> Input mouth area = $S_s$	$F_0 S_s \equiv J_s (= J_D \cdot \text{<at max temperature>}).$ *
<b>Max Heat Output</b> (by ventilation) $= J_{DR} + J_{DV} \doteq J_{DV} \equiv J_D$ $J_{DR} \doteq 0$ radiation loss	$J_{DV} = \oint dS(s) \kappa_v(s) (T(s) - T_E)$ $\equiv S_B \kappa_v(s') (T(s') - T_E)$ $\equiv K (T_{max} - T_E). \rightarrow K = J_s / (T_m - T_E).$
<b>Max temperature</b>	$T_H(\infty) = J_s / K.$
<b>Heat up time constant :</b>	$\tau = C_H / K. \quad \ll C_H = \text{heat load } C_L + \text{heat box } C_B \gg$

(3) **Time Response of Heater Temperature.**

Solar heater is completely free, but exception is **not quick action** in small solar input.

This section is very important to get better design parameters of solar heater you wish.

(a) **heat capacity of heater × temperature rise/sec = {heat-input – heat-output}/sec**

$$C_H (dT_H/dt) = J_S - J_D \dots \dots \dots \ll \text{heat account equation} \gg.$$

(b)  $J_D \doteq K \langle T_H - T_0 \rangle \dots \dots \dots (2)(a) \dots \dots \dots \ll \text{see more details in (4)} \gg$

**☞ : Linear heat loss approximation (air ventilation loss in surface of outer box).**

(c)  $J_S = (\text{constant average assumption})$ .

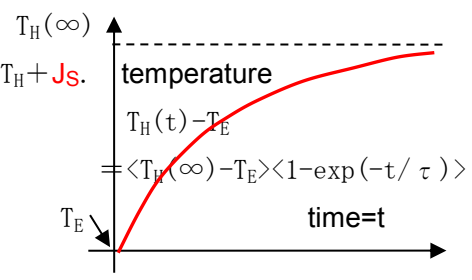
$$C_H (dT_H/dt) = J_S - K \langle T_H - T_0 \rangle = -KT_H + (KT_0 + J_S) \equiv -KT_H + J_S.$$

$$dT_H/dt = - (K/C_H) T_H + (J_S/C_H) \equiv -T_H / \tau + T'_H(0).$$

$$\rightarrow dT_H/dt + T_H / \tau = T'_H(0).$$

$$\rightarrow d [T_H \exp(t / \tau)] / dt = T'_H(0) \exp(t / \tau).$$

$$\rightarrow T_H \exp(t / \tau) = \tau T'_H(0) \exp(t / \tau) + C.$$



(d) **Heat up time function.**

$$T_H(t) = T_E + (T_H(\infty) - T_E) \langle 1 - \exp(-t / \tau) \rangle. \rightarrow t(T) = -\tau \ln(1 - \langle T - T_E \rangle / \langle T_H(\infty) - T_E \rangle).$$

example)  $\langle T - T_E \rangle / \langle T_H(\infty) - T_E \rangle = 100/120, \rightarrow t(T) = -\tau \ln(1/6) = 1.8 \tau$

$\langle T - T_E \rangle / \langle T_H(\infty) - T_E \rangle = 100/105, \rightarrow t(T) = -\tau \ln(5/105) = 3 \tau$

\* **<in the below, yellow portion is authors estimation>**

(e) **Heat up time constant :  $\tau = C_H / K (= C_H \langle T_H(\infty) - T_E \rangle / J_S)$ .**

example)  $\tau \equiv C_H / K \sim (12 \text{KJ}/^\circ\text{C} + 8 \text{KJ}/^\circ\text{C}) / (7 \text{W}/^\circ\text{C}) = 0.8 \text{h}, \rightarrow 3 \tau = 2.4 \text{h}.$

$\ll \text{☞ : } C_H = 12 \text{KJ}/3 \text{kg-water} \gg$

: Larger heat load =  $C_H$  takes longer heat up time =  $3 \tau$ .

$C_H =$  heat capacity of {heat load  $\equiv C_L$  + heat box  $\equiv C_B$ }.

☞ : caution,  $\tau$  is not heat up time you want (such as  $T = 100^\circ\text{C}$ ), but is **time of final highest temperature. Stronger  $J_S$  can make shorter time, and larger  $C_H$  makes longer time..**

(f) **max temperature :  $T_H(\infty) = J_S / K (= J_S \tau / C_H)$ .**

$T_H(\infty) = \tau T'_H(0) = (C_H / K) (J_S / C_H) = J_S / K \sim 700 \text{W} / (8 \text{W}/^\circ\text{C}), 800 \text{W} / (9 \text{W}/^\circ\text{C}) \sim 90^\circ\text{C}.$   $\ll \langle T_H(t=0) \equiv \rangle \gg.$

Larger air ventilation =  $K$  makes lower max temperature =  $T_H(\infty)$ .

☞ :  $K = \tau / C_H = J_S / \langle T_H(\infty) - T_E \rangle \doteq 7 \sim 20 \text{W}/^\circ\text{C} ?$ .

(g)  $T'_H(0) = J_S / C_H.$

(4) As for the parameter "K" in "(3)(b)  $J_D \doteq K(T_H - T_0)$  . . . . . (2)(a)" .

This K is most important parameter determining **max temperature of heat box**:  $T_H(\infty) = J_s/K$  and heat up time :  $\tau = C_H/K$  . K is originally due to following form.

[1](2)(a) **surface air ventilation loss** : {  $\kappa_v = 7W/m^2C$  in room but larger in field with wind }

$$J_{vbt} = S_{bt} \kappa_v (T_{bt} - T_E) = (0.6m \times 0.6m) \times 2.0 \times 7W/m^2C \times 90C \sim 450W$$

$$J_{vbs} = S_{bs} \kappa_v (T_{bs} - T_E) = (0.6m \times 0.6m + 4 \times 0.6 \times 0.3) \times 2.0 \times 7W/m^2C \times 20C \sim 300W$$

The strict form is as follows. Then a kernel problem is **wind dependent parameter** =  $\kappa_v$ .  
 $J_{vs} = \oint dS (s) \kappa_v(s) (T(s) - T_E) = S_T \langle \kappa_v(s') (T(s') - T_E) \rangle \equiv K (T_m - T_E)$   
 = total surface area  $\times \kappa_v(s')$   $\times$  < surface s' temperature at max box  $T_m$  - minimum  $T_E$  >

\*  $dS (s)$  = differential surface area of local portion s.

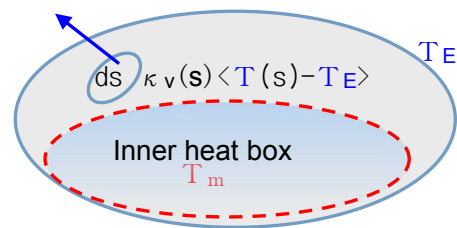
$$\oint dS (s) = S_T \langle \text{total area of surface} \rangle.$$

\*  $\kappa_v(s) = \kappa_v$  at local surface portion s.

$$\kappa_v(s) = 7W/m^2C \times (1 \sim 3?) \text{ by wind.}$$

\* maximum  $T_s =$  maximum T of surface portion = s.

\* Minimum T = environmental air  $T_E$ .



\*  $K \equiv S_T \langle \kappa_v(s') (T(s') - T_E) \rangle / (T_m - T_E) = J_s / (T_m - T_E)$ .  
 $(T(s') - T_E) / (T_m - T_E) \doteq 0.2 \sim 1$ .  $\kappa_v(s) = 1 \sim 3? \times 7W/m^2C$  by wind.

example calculation)

$$S_T = 1.44m^2; (T(s') - T_E) / (T_m - T_E) \doteq 0.4. \quad \kappa_v(s) \doteq 2 \times 7W/m^2C \text{ by wind.}$$

$$\rightarrow K \equiv S_T \langle \kappa_v(s') (T(s') - T_E) \rangle / (T_m - T_E) \doteq 8.1W/C, \text{ or } 750W / (100 - 10) \doteq 8.3W/C.$$

(5) Time response in measured value. 2 0 1 5 - 1 - 1 4



environmental condition  
**air temperature**  
 $T_E \sim 10C$   
**wind velocity**  
 $V_W \sim$  almost nothing.  
**solar angle**  
 almost perpendicular  
 by rotational base operation

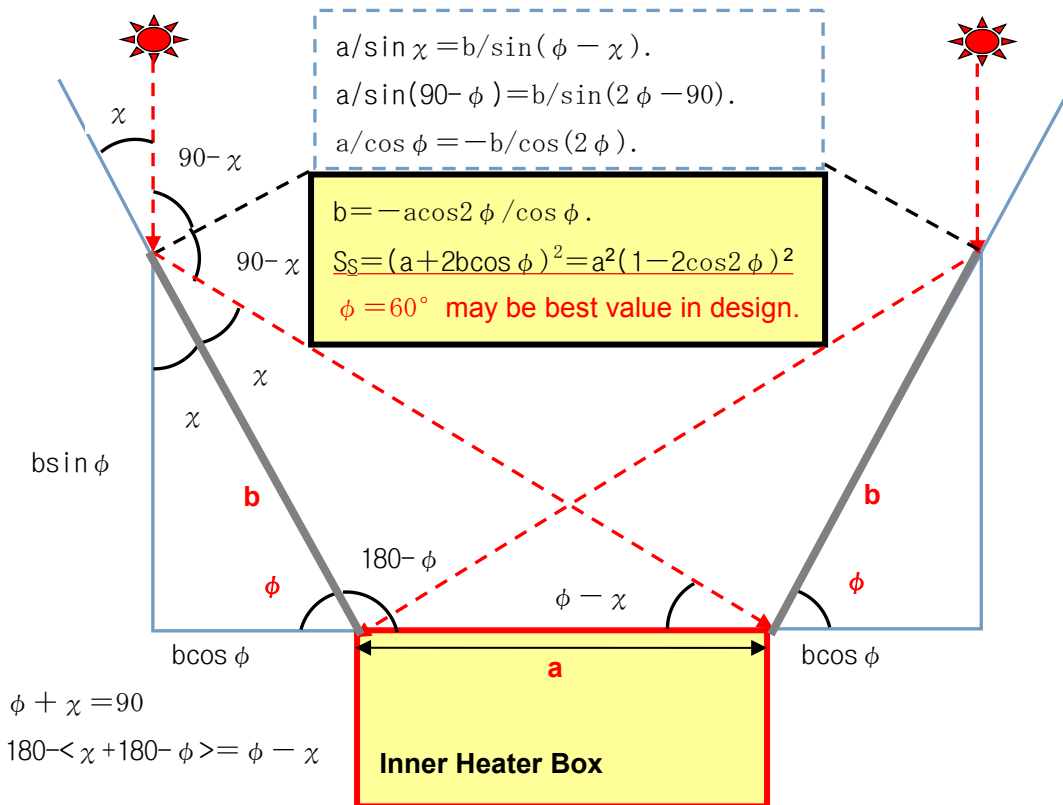
Note above **100C** is very calm boiling point, but not strong stirring due to  $J_s \doteq J_D$ .

The experiment is most cold season July in northern hemisphere latitude 43(Tokyo).

Therefore another warmer season would be more quick heat up time.

**[ 2 ] : Calculation on Reflector Angle and Reflector Size.**

How to determine design parameters of {**a**=box size,**b**=reflector length,and  $\phi$ } in order to optimize **effective heat input mouth area** =  $S_s(\phi ; a, b)$  ?.



**\* optimization on larger mouth vs less volume.**

Getting **boiling temperature** for 2 kettles is aim(as for author),so it should be large..

While,**plywood** area is 180cm×90cm,which should be used without loss.And also

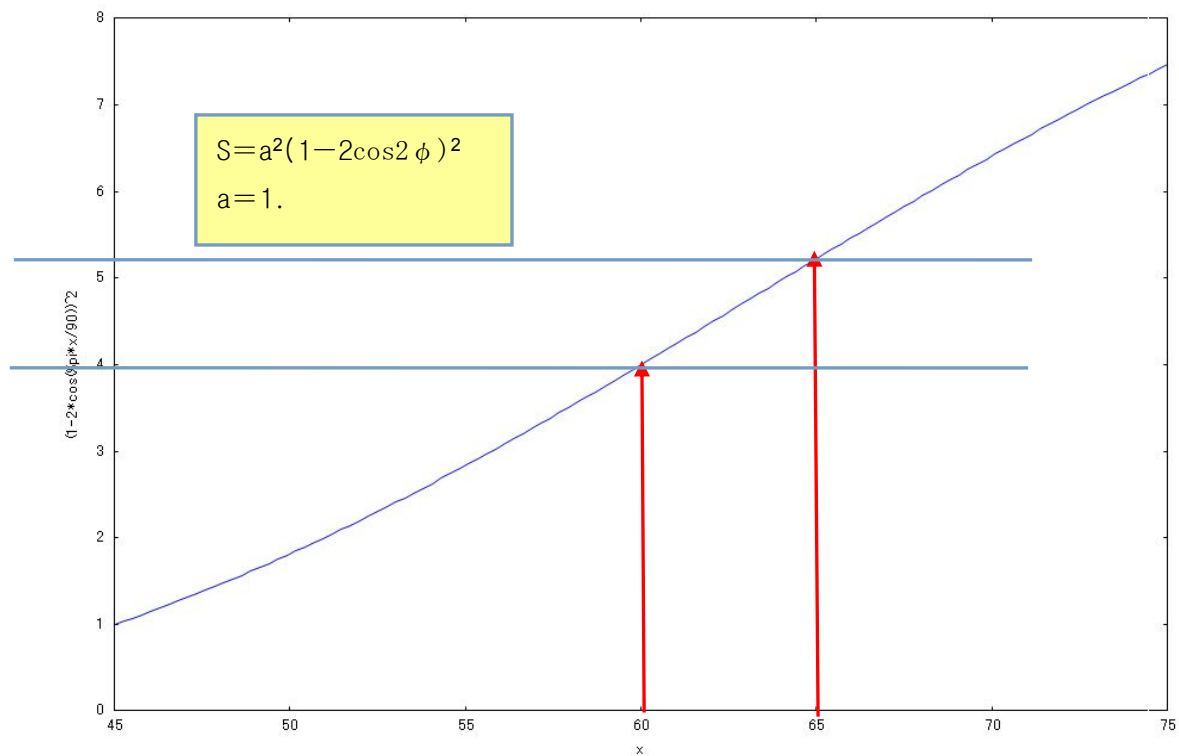
the solar set must not be damaged by **strong wind**,so it should be small.

Then author concluded  $\phi = 60^\circ$  , **a**=box size=45cm,,**b**=reflector length=45cm.

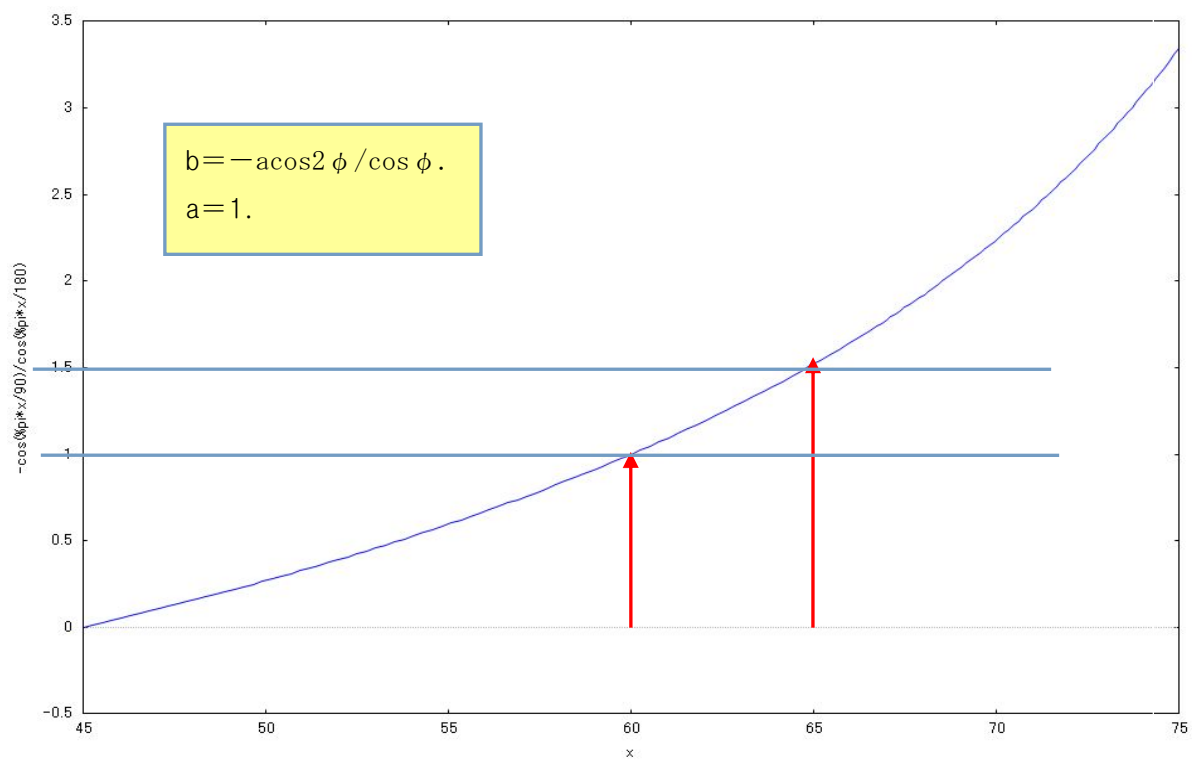
See plot 2.  $\phi = 60^\circ$  is with **b**=45cm、 while  $\phi = 65^\circ$  is with **b**=67.5m、

The latter **b**=67.5m becomes larger while mouth increasing is 25%<See plot 1.>.

1. `plot2d((1-2*cos(4*x*pi/360))^2, [x, 45, 75]);`

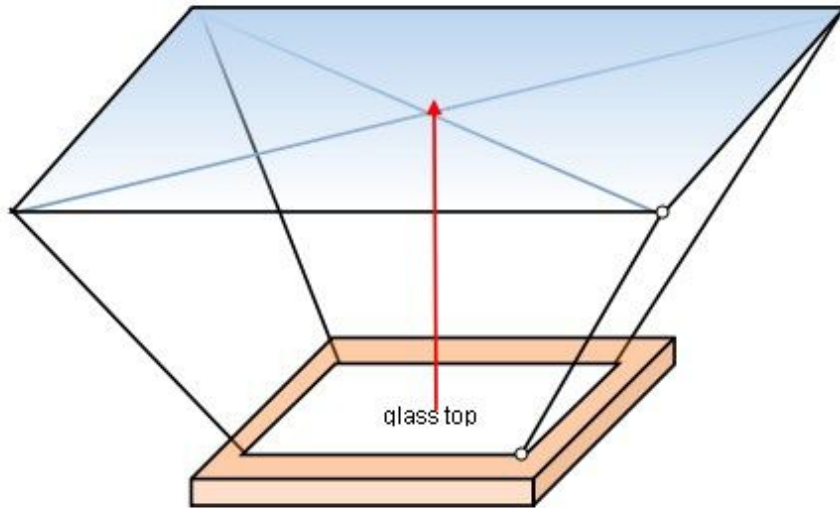


2. `plot2d(-cos(4*x*pi/360)/cos(2*x*pi/360), [x, 45, 75]);`

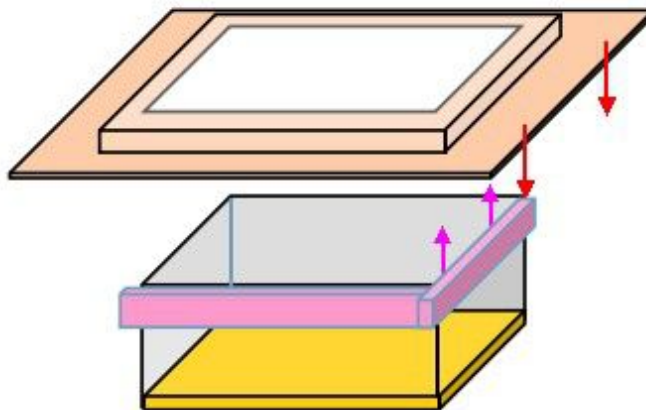


over whole view for fabrication sequence <bottom to top>. 2014/11/20

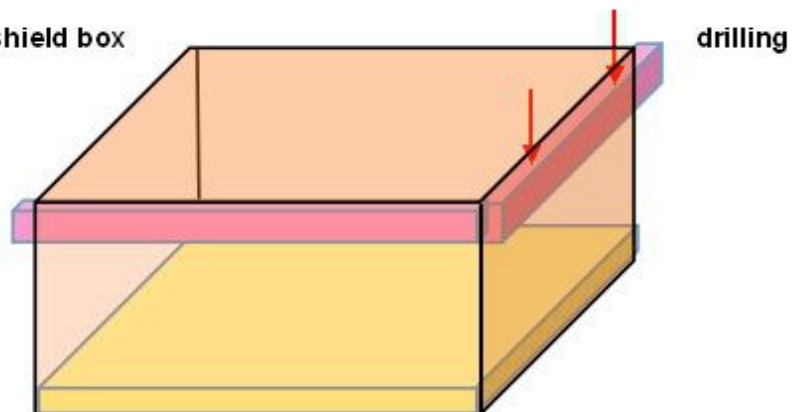
### Reflectors



### inner heat box junction

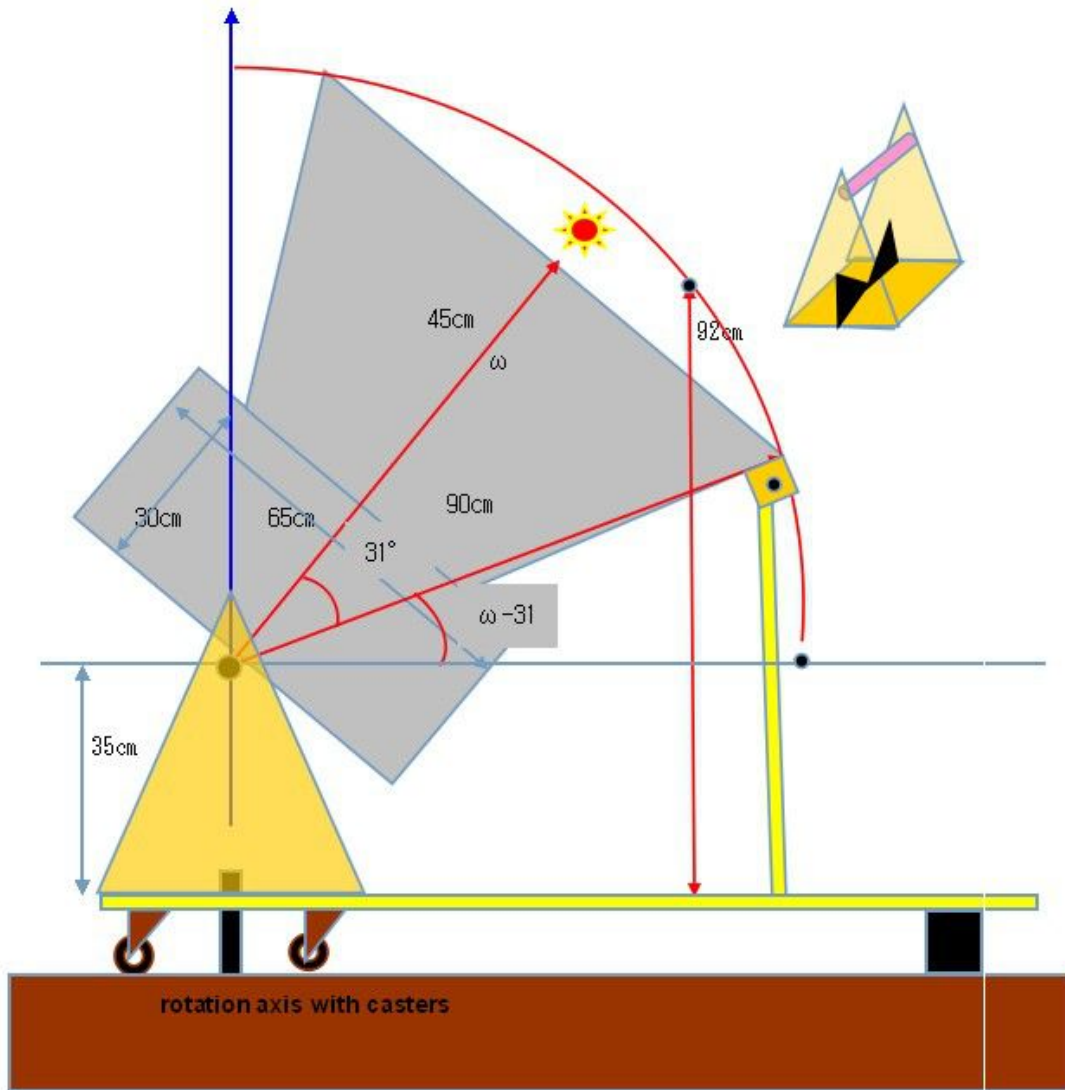


### outer shield box





Rotational base.



$\omega =$	30	40	50	60	70
$Y=90\sin(\omega - 31)$	0	14	29.3	43.6	57
L (stick)	35	49	64.3	79	92