

**Appendix\_1: Dynamics in Rotational Coordinate.**

2013-3-27,31,4/18

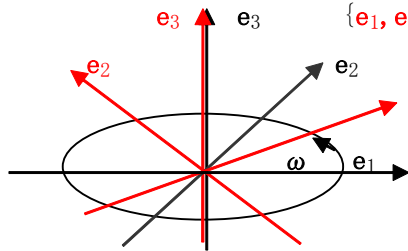
(1)coordinates:

$\{e_1, e_2, e_3\}$  is inertia fixed coordinate(IC).

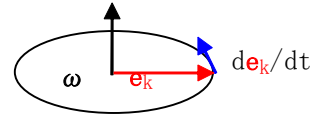
$\{e_1, e_2, e_3\}$  is non-inertia rotational coordinate(NC)

with **angular velocity vector**  $\equiv \omega$ .

Then note that  $d\mathbf{e}_k/dt = \omega \times \mathbf{e}_k$ .



$e_1$



(2)position vector:  $\mathbf{R} = \sum_{k=1}^3 R_k \mathbf{e}_k$ .

(3)velocity vector:

$$\begin{aligned} \mathbf{V} &= d\mathbf{R}/dt = \sum_{k=1}^3 (dR_k/dt) \mathbf{e}_k + \sum_{k=1}^3 R_k (d\mathbf{e}_k/dt) = \sum_{k=1}^3 (dR_k/dt) \mathbf{e}_k + \sum_{k=1}^3 R_k (\omega \times \mathbf{e}_k) \\ &= \sum_{k=1}^3 V_k \mathbf{e}_k + (\omega \times \mathbf{R}) = \mathbf{V} + (\omega \times \mathbf{R}). \end{aligned}$$

(4)acceleration vector  $\alpha$ :

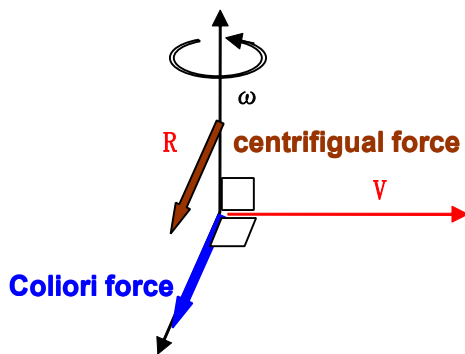
$$\begin{aligned} \alpha &\equiv d\mathbf{V}/dt = \sum_{k=1}^3 (dV_k/dt) \mathbf{e}_k + \sum_{k=1}^3 V_k (d\mathbf{e}_k/dt) + (d\omega/dt \times \mathbf{R} + \omega \times d\mathbf{R}/dt) \\ &= \sum_{k=1}^3 (dV_k/dt) \mathbf{e}_k + (\omega \times \mathbf{V}) + d\omega/dt \times \mathbf{R} + \omega \times (\mathbf{V} + (\omega \times \mathbf{R})) \\ &= d\mathbf{V}/dt + 2(\omega \times \mathbf{V}) + \omega \times (\omega \times \mathbf{R}) + d\omega/dt \times \mathbf{R} \quad \text{"in climate science, } d\omega/dt = 0". \end{aligned}$$

$$\mathbf{f} \equiv m d\mathbf{V}/dt = m d\mathbf{V}/dt + 2m(\omega \times \mathbf{V}) + m\omega \times (\omega \times \mathbf{R}).$$

(5)  $\mathbf{f} \equiv m d\mathbf{V}/dt$ ,  $\mathbf{f} \equiv 0$ , **free motion** in IC and Coliolis force in NC.

$$\mathbf{f} \equiv m d\mathbf{V}/dt \quad \mathbf{f} \equiv 0. \rightarrow 0 = m d\mathbf{V}/dt + 2m(\omega \times \mathbf{V}) + m\omega \times (\omega \times \mathbf{R}).$$

$$\rightarrow m d\mathbf{V}/dt = -2m(\omega \times \mathbf{V}) - m\omega \times (\omega \times \mathbf{R}) \equiv \text{Coliori force} + \text{centrifugal force}.$$

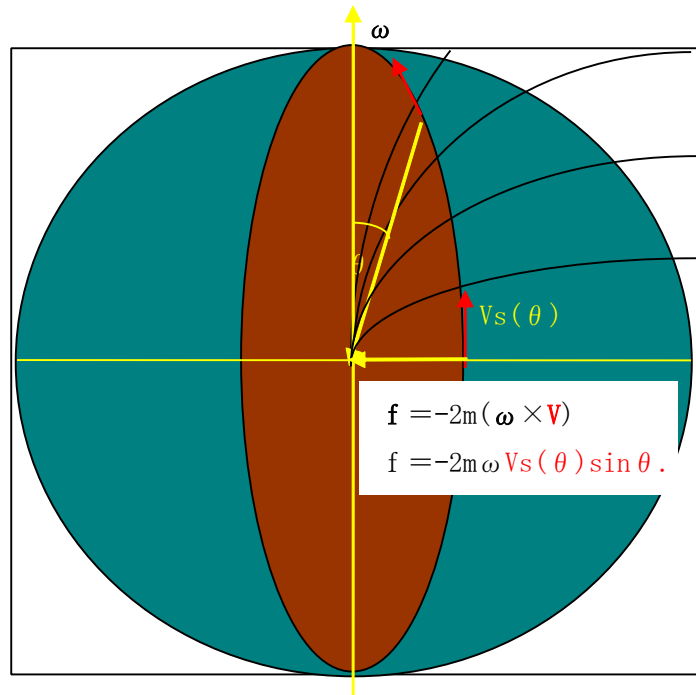


\* Earth centrifugal force(R=radius)

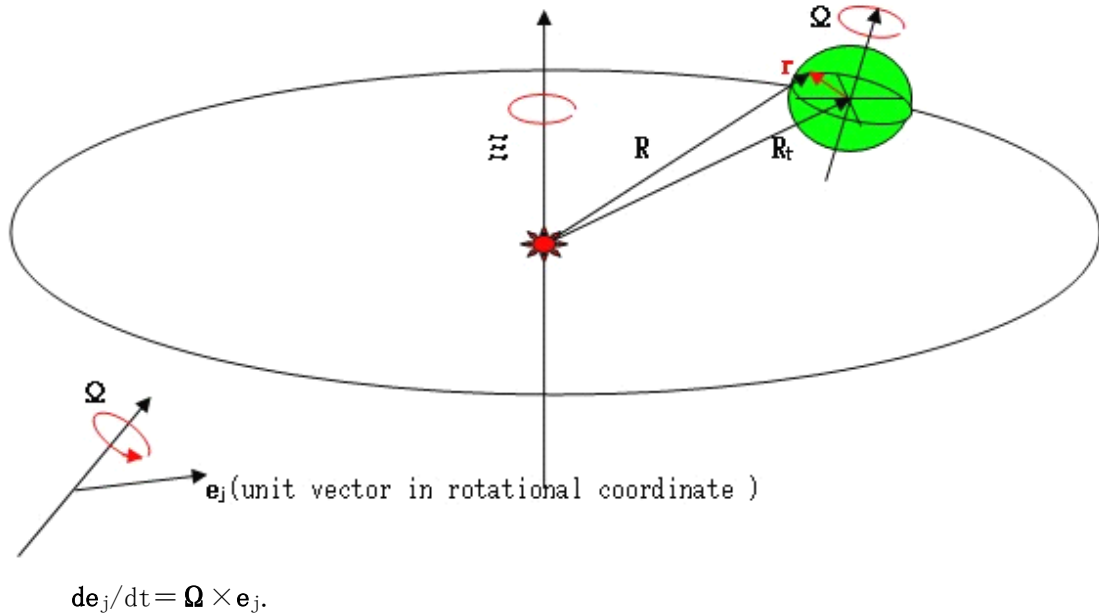
$$= -\rho \omega \times (\omega \times \mathbf{R}) \ll \rho g$$

=gravity force,so it(0.34%) is

neglegible in geophysics.



(6) **Orbital Force**



Traveling= $\mathbf{R}_t$  and self-rotational coordinate= $\mathbf{r}$ :

$$\mathbf{R} \equiv \mathbf{r} + \mathbf{R}_t.$$

$$\mathbf{V} \equiv d\mathbf{R}/dt = d\mathbf{r}/dt + d\mathbf{R}_t/dt = d(r_j \mathbf{e}_j)/dt + d\mathbf{R}_t/dt = \mathbf{v} + (\boldsymbol{\Omega} \times \mathbf{r}) + d\mathbf{R}_t/dt.$$

$$d(r_j \mathbf{e}_j)/dt = \mathbf{e}_j dr_j/dt + r_j d\mathbf{e}_j/dt = \mathbf{e}_j dr_j/dt + r_j (\boldsymbol{\Omega} \times \mathbf{e}_j).$$

$$= \mathbf{v} + (\boldsymbol{\Omega} \times \mathbf{r}).$$

$$\mathbf{A} \equiv d^2\mathbf{R}/dt^2 = \mathbf{e}_j d^2 r_j/dt^2 + d\mathbf{e}_j/dt \cdot dr_j/dt + dr_j/dt (\boldsymbol{\Omega} \times \mathbf{e}_j) + r_j (\boldsymbol{\Omega} \times d\mathbf{e}_j/dt) + d\mathbf{R}_t^2/dt^2$$

$$= \mathbf{e}_j d^2 r_j/dt^2 + 2 dr_j/dt (\boldsymbol{\Omega} \times \mathbf{e}_j) + r_j (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{e}_j)) + d\mathbf{R}_t^2/dt^2.$$

$$= \mathbf{e}_j d^2 r_j/dt^2 + 2(\boldsymbol{\Omega} \times \mathbf{v}) + (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})) + d\mathbf{R}_t^2/dt^2.$$

$$\boldsymbol{\alpha} \equiv \mathbf{e}_j d^2 r_j/dt^2 = \mathbf{A} - 2(\boldsymbol{\Omega} \times \mathbf{v}) - (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})) - d\mathbf{R}_t^2/dt^2.$$

$$d\mathbf{R}_t^2/dt^2 = \mathbf{E}_j d^2 R_{tj}/dt^2 + 2(\boldsymbol{\Xi} \times \mathbf{v}_t) + (\boldsymbol{\Xi} \times (\boldsymbol{\Xi} \times \mathbf{R}_t))$$

$$\mathbf{e}_j d^2 r_j/dt^2 = \mathbf{A} - 2(\boldsymbol{\Omega} \times \mathbf{v}) - (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})) - \underline{2(\boldsymbol{\Xi} \times \mathbf{v}_t) + (\boldsymbol{\Xi} \times (\boldsymbol{\Xi} \times \mathbf{R}_t))}.$$

$\boldsymbol{\Xi} = \boldsymbol{\Omega}/365$ , while  $R_t = \text{earth radius} \times 23500$ . 1:24h-23500:8760h// velocity ratio=

1/24 : 23500/8760 = 1 : 64.4 times,  $\boldsymbol{\Xi} \mathbf{V} : \boldsymbol{\Omega} \mathbf{v} = 64.4/365 : 1 = 0.176 : 1$

**Orbital coliori force** is smaller than spin coliori one, but might be **not negligible**.

## Appendix\_2:

**Large space and time scale view** by scaling transformation(Reinolds analogy low) .

So called Chaos in fluid equation is due to friction term  $\mu \nabla^2 \mathbf{u}$  , which causes long term weather prediction difficult.However a method could overcome the difficulty.

Fluid equation could be transformed into new space and time variables $\{x',t'\}$  from  $\{x,t\}$  by  $\{x \equiv Lx', t \equiv (L/U)t', \rho \equiv \rho' ; m \equiv L^3 m', \Omega \equiv \Omega' \}$

$$\begin{aligned} u &\equiv \partial x / \partial t = \partial (Lx')_x / \partial ((L/U)t') = u \partial x' / \partial t' = Uu' . \\ \alpha &\equiv \partial u / \partial t = \partial (Uu') / \partial ((L/U)t') = (U^2/L) \partial u' / \partial t' = (U^2/L) \alpha' . \\ P &\equiv f/x^2 = m \alpha / x^2 = L^3 m' (U^2/L) \alpha' / (Lx')^2 = U^2 m' \alpha' / x'^2 = U^2 p' . \\ \partial P / \partial x &= (U^2/L) \partial p' / \partial x' . \\ \rho &\equiv m/x^3 = L^3 m' / (Lx')^3 = m' / x'^3 = \rho' . \\ \partial^2 u / \partial x^2 &= \partial^2 (Uu') / \partial (Lx')^2 = (U/L^2) \partial^2 u' / \partial x'^2 \\ -2 \rho \Omega \times \mathbf{u} &= (U) 2 \rho' \Omega' \times \mathbf{u}' \end{aligned}$$

$$\begin{aligned} \rho \{ \partial \mathbf{u} / \partial t + \mathbf{u} \cdot \text{grad} \mathbf{u} \} &= -\text{grad} P + \mu \nabla^2 \mathbf{u} - 2 \rho \Omega \times \mathbf{u} + \rho \mathbf{g} . \\ \rho' \{ (U^2/L) \partial \mathbf{u}' / \partial t' + (U^2/L) \mathbf{u}' \cdot \text{grad}' \mathbf{u}' \} \\ &= -(U^2/L) \text{grad}' P' + \mu (U/L^2) \nabla'^2 \mathbf{u}' - (U) 2 \rho' \Omega' \times \mathbf{u}' + \rho' \mathbf{g} . \end{aligned}$$

$$\begin{aligned} \rho' \{ \partial \mathbf{u}' / \partial t' + \mathbf{u}' \cdot \text{grad}' \mathbf{u}' \} \\ &= -\text{grad}' P' + (\mu / UL) \nabla'^2 \mathbf{u}' - 2 \rho' (L/U) \Omega' \times \mathbf{u}' + \rho' (L/U^2) \mathbf{g} \\ &= -\text{grad}' (P/U^2) + (\underline{\mu / UL}) \nabla'^2 \mathbf{u}' - 2 \rho' (\underline{L/U}) \Omega' \times \mathbf{u}' + \rho' (\underline{L/U^2}) \mathbf{g} . \end{aligned}$$

Then taking  $U=1, L=\text{larger}$  could make  $(\mu / UL) \equiv \mu'$  smaller(almost nothing friction force).  
grad' u', Pressure gradient, Coriolis force and gravity become larger.

This is to make fluid equation more causal one in **large space and time scale view**.

This might be a certification for global climate model calculation.

example) :

$$1\text{m (man size)} \rightarrow 6 \times 10^6 \text{m (earth size)} \Rightarrow U \equiv 1 ; L \equiv 10^6 .$$

$$\mu = 1.307 \times 10^{-3} \text{Ns/m}^2 \rightarrow \mu' = \mu / UL \doteq 0 .$$

$$\rho' \{ \partial \mathbf{u}' / \partial t' + \mathbf{u}' \cdot \text{grad}' \mathbf{u}' \} = -\text{grad}' (P') - 2 \rho' (\underline{L/U}) \Omega' \times \mathbf{u}' + \rho' (\underline{L/U^2}) \mathbf{g} .$$

In order to be **complete dynamic equation** ,left term need  $+\mathbf{u} \{ \partial \rho / \partial t + \mathbf{u} \cdot \text{grad} \rho \}$  for force by density  $\rho'$  change.

**Appemdex\_3:**

**spiral westery flow in the stationaly equation solution.**

2013/3/23, 4/05

$$\rho (\mathbf{v} \cdot \nabla \mathbf{v}) = -2 \rho \boldsymbol{\Omega} \times \mathbf{v} - \nabla p + \rho \mathbf{g}.$$

Above **non-linear equation** could not be solved by analytical way, then author assumed following spirral flow solution and derived the correspondent **pressure force**.

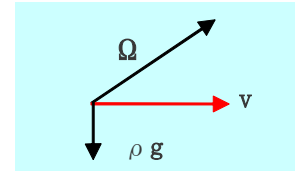
**(1)The most simple solution.**

Assuming  $\rho (\mathbf{v} \cdot \nabla \mathbf{v}) = 0; \nabla p = 0$  yield westery flow.

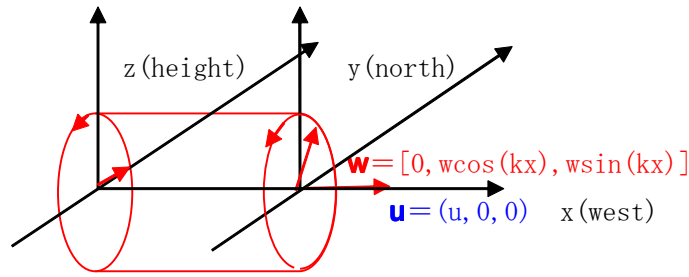
$$0 = -2 \rho \boldsymbol{\Omega} \times \mathbf{v} + \rho \mathbf{g}.$$

$\mathbf{v}$  could be a westery jet stream by downward force =  $\rho \mathbf{g}$ ,

In the below, let's assume spirral flow component  $\mathbf{w} \equiv (0, w \cos(kx), w \sin(kx))$ .



**(2)Elliptic Spirral Pipe Line Flow solution in fluid dyanamics.**

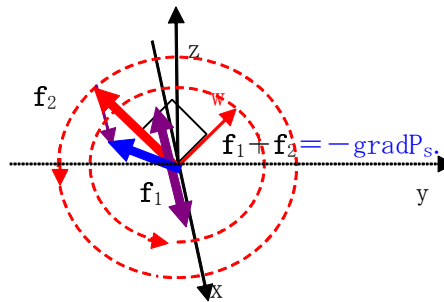


$$\mathbf{u} = (u, 0, 0).$$

$$\mathbf{w} \equiv (0, w \cos(kx), w \sin(kx)).$$

$$* \mathbf{v} = \mathbf{u} + \mathbf{w} = (u, w \cos(kx), w \sin(kx))$$

$$\left( \begin{array}{l} \text{grad } v_1 = (0, 0, 0) \\ \text{grad } v_2 = (-kw \sin(kx), 0, 0) \\ \text{grad } v_3 = (kw \cos(kx), 0, 0) \end{array} \right)$$



$$\rho \mathbf{v} \cdot \text{grad } \mathbf{v} = \rho u w k [0, -\sin(kx), \cos(kx)] \equiv \mathbf{F}. \quad \langle\langle \rho \text{ driving force} \rangle\rangle$$

The required force for space derivative accelation is spirral with axis x(west-ward).

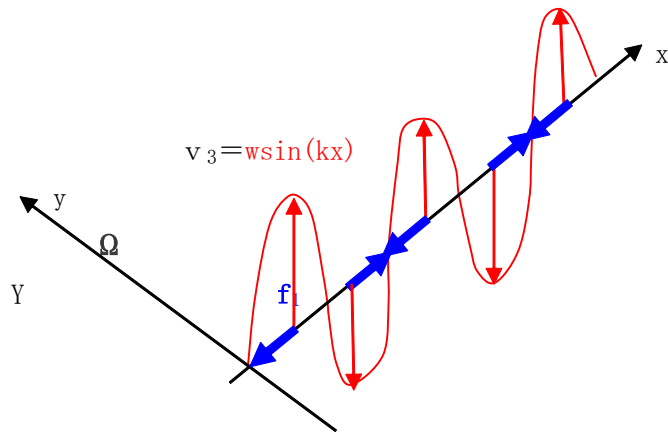
$$\rho \mathbf{v} \cdot \text{grad } \mathbf{v} = -2 \rho \boldsymbol{\Omega} \times (\mathbf{u} + \mathbf{w}) + \rho \mathbf{g} - \text{grad } P_s = -2 \rho \boldsymbol{\Omega} \times \mathbf{w} - \text{grad } P_s = \mathbf{F}.$$

$$-\text{grad } P_s = 2 \rho \boldsymbol{\Omega} \times \mathbf{w} + \mathbf{F} = 2 \rho \boldsymbol{\Omega} \times \mathbf{w} + \rho u w k [0, -\cos(kx), \sin(kx)]$$

$-\text{grad } P_s = [2 \rho \Omega w \sin(kx), 0, 0] + \rho u w k [0, -\cos(kx), \sin(kx)] \equiv \mathbf{f}_1 + \mathbf{f}_2$   
**= "vibration force by coliori on x axis" + "circulating force in yz plane"**

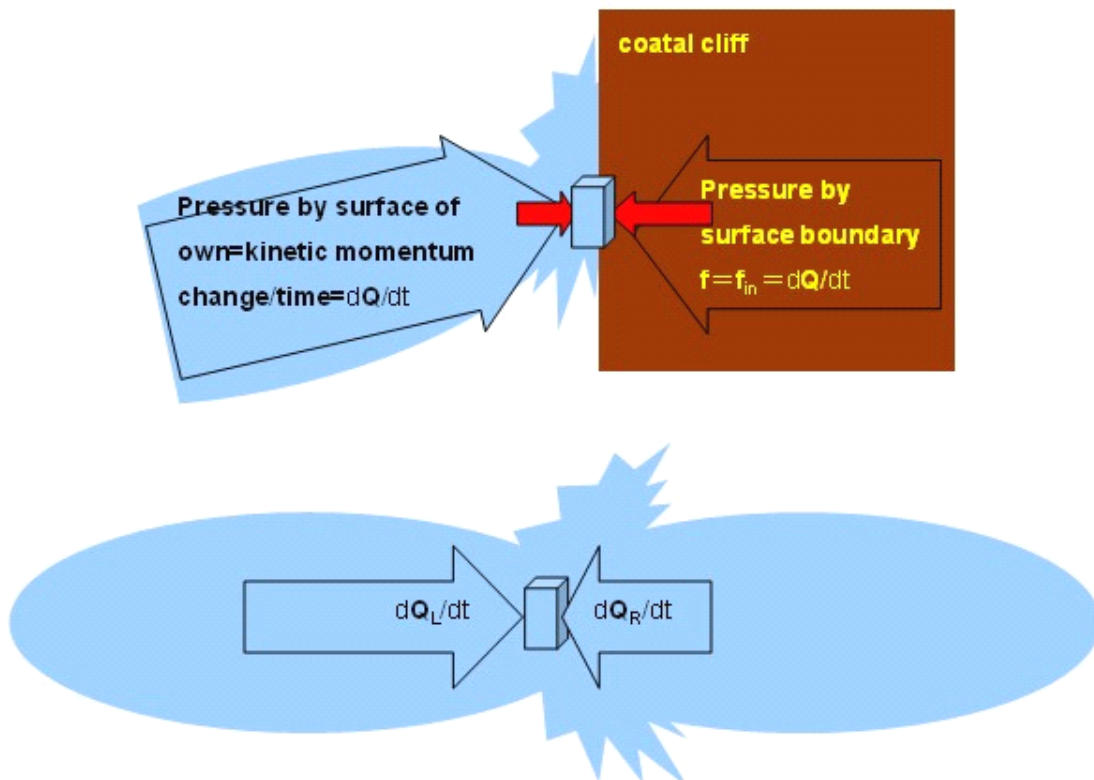
$f_1 \equiv$  "vibration force by coliori on x axis"

=updown velocity in z axis causes coliori force for the vibration in x axis.



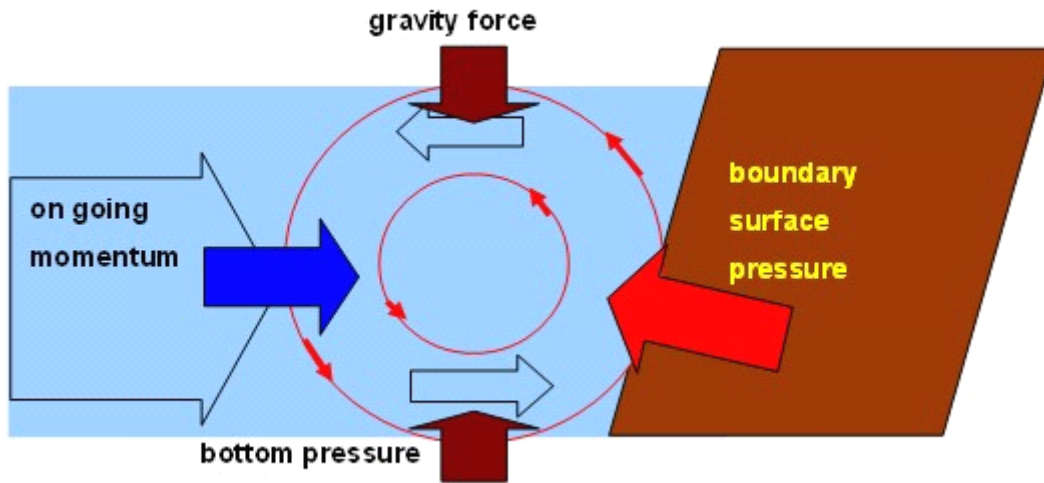
$f_2 \equiv$  "circulating force in yz plane"

**Pessure**(plain force) on fluid has only two aspects of **surface boundary** and **surface of own**.

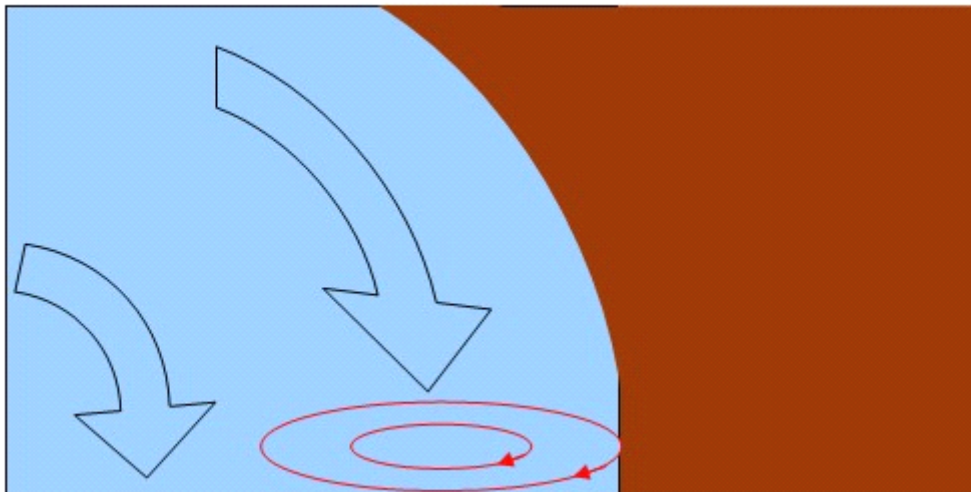


$f_2 \equiv$  "circulating force in yz plane"

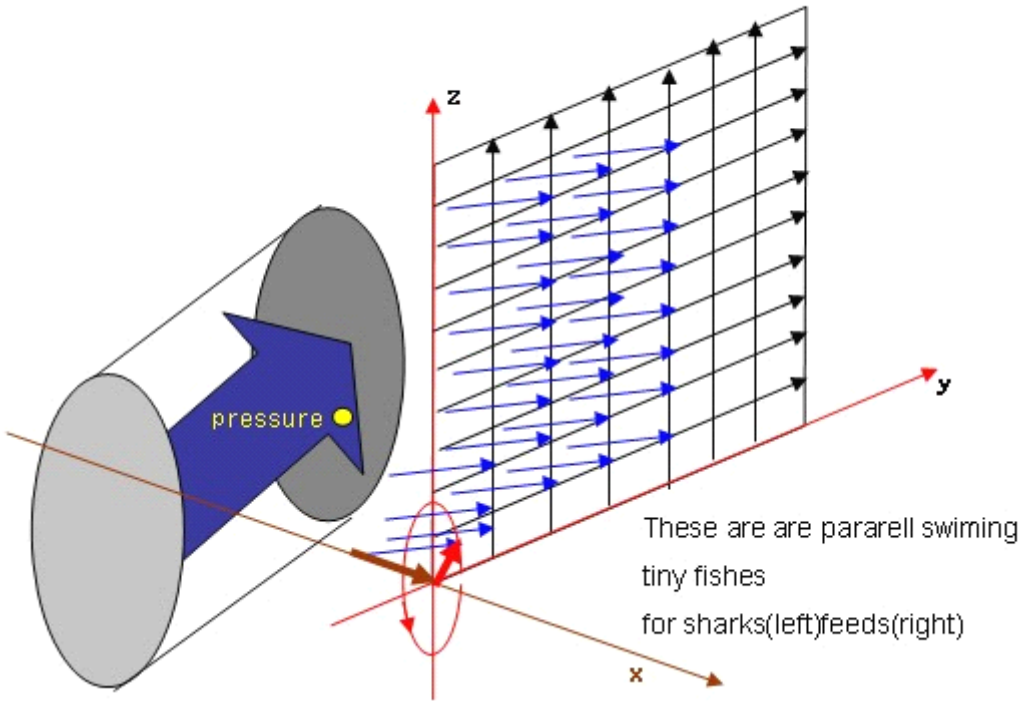
<<side view>>



<top view>>



"The image of the 1 dimensional solution"



### (3) Elliptic Rotation Solution:

Reference:沿岸海域の物質分散(4)-潮流楕円の鉛直構造-

柳哲雄、西井正樹、樋口明生、京大防災研究所年報代26号 B-2(S58.4).

[http://www.dpri.kyoto-](http://www.dpri.kyoto-u.ac.jp/dat/nenpo/no26/26b2/a26b2p39.pdf#search='%E6%B2%BF%E5%B2%B8%E6%B5%B7%E5%9F%9F%E3%81%AE%E7%89%A9%E8%B3%AA%E5%88%86%E6%95%A3%284%29%E6%BD%AE%E6%B5%81%E6%A5%95%E5%86%86%E3%81%AE%E9%89%9B%E7%9B%B4%E6%A7%8B%E9%80%A0')

[u.ac.jp/dat/nenpo/no26/26b2/a26b2p39.pdf#search='%E6%B2%BF%E5%B2%B8%E6%B5%B7%E5%9F%9F%E3%81%AE%E7%89%A9%E8%B3%AA%E5%88%86%E6%95%A3%284%29%E6%BD%AE%E6%B5%81%E6%A5%95%E5%86%86%E3%81%AE%E9%89%9B%E7%9B%B4%E6%A7%8B%E9%80%A0'](http://www.dpri.kyoto-u.ac.jp/dat/nenpo/no26/26b2/a26b2p39.pdf#search='%E6%B2%BF%E5%B2%B8%E6%B5%B7%E5%9F%9F%E3%81%AE%E7%89%A9%E8%B3%AA%E5%88%86%E6%95%A3%284%29%E6%BD%AE%E6%B5%81%E6%A5%95%E5%86%86%E3%81%AE%E9%89%9B%E7%9B%B4%E6%A7%8B%E9%80%A0')

$$\mathbf{u} = (u, 0, 0).$$

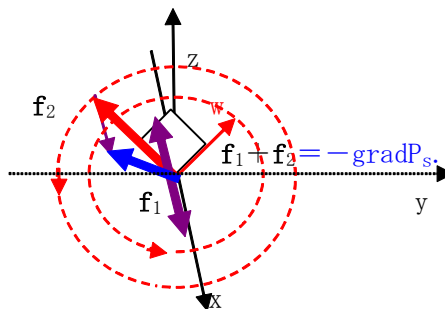
$$\mathbf{w} \equiv (0, Y \cos(kx), Z \sin(kx)).$$

$$\mathbf{v} = \mathbf{u} + \mathbf{w} = (u, Y \cos(kx), Z \sin(kx)).$$

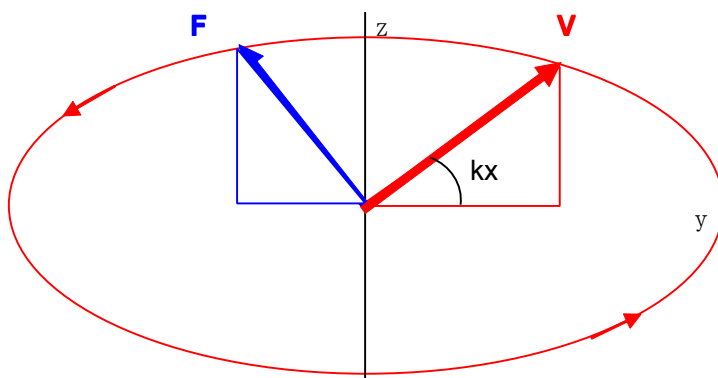
$$\text{grad } v_1 = (0, 0, 0)$$

$$\text{grad } v_2 = (-kY \sin(kx), 0, 0)$$

$$\text{grad } v_3 = (kZ \cos(kx), 0, 0)$$



$$\rho \mathbf{v} \cdot \text{grad } \mathbf{v} = \rho u k [0, -Y \sin(kx), Z \cos(kx)] \equiv \mathbf{F}.$$



The required force for space derivative acceleration is spiral with axis x(west-ward).

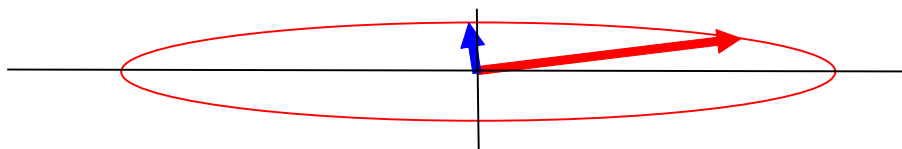
$$\rho \mathbf{v} \cdot \text{grad } \mathbf{v} = -2 \rho \boldsymbol{\Omega} \times (\mathbf{u} + \mathbf{w}) + \rho \mathbf{g} - \text{grad } P_s = -2 \rho \boldsymbol{\Omega} \times \mathbf{w} - \text{grad } P_s.$$

$$\mathbf{F} \equiv \rho u k [0, -Y \sin(kx), Z \cos(kx)] = -2 \rho \boldsymbol{\Omega} \times \mathbf{w} - \text{grad } P_s.$$

$$-\text{grad } P_s = 2 \rho \boldsymbol{\Omega} \times \mathbf{w} + \rho u k [0, -Y \sin(kx), Z \cos(kx)]$$

$$-\text{grad } P_s = [2 \rho \boldsymbol{\Omega} Z \cos(kx), 0, 0] + \rho u k [0, -Y \sin(kx), Z \cos(kx)] \equiv \mathbf{f}_1 + \mathbf{f}_2.$$

"plate wave pulling an pushig in beach slope may be driven by flowing extreme elliptic"

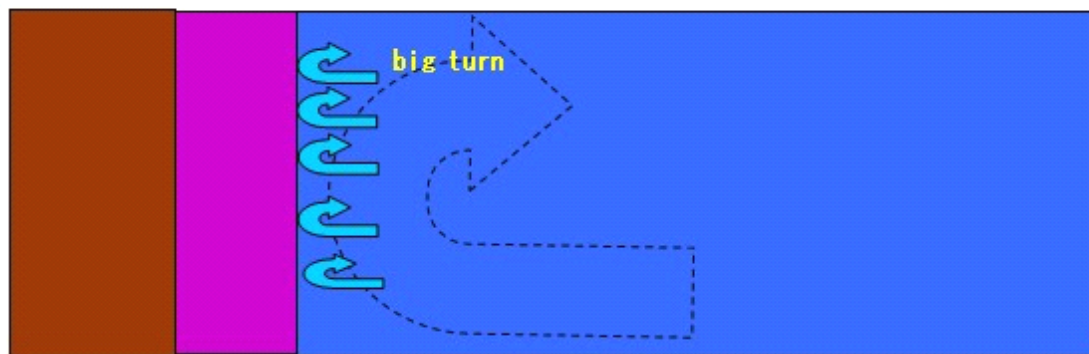
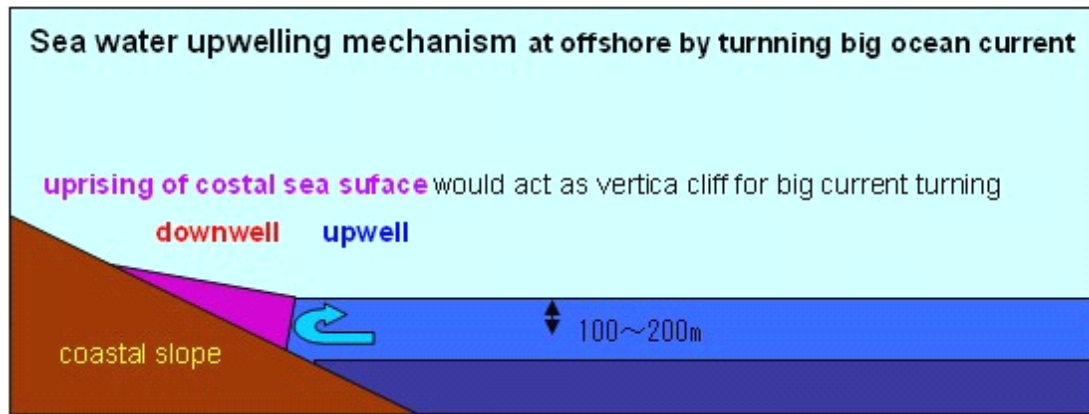


Updown force is due to boundary pressure and gravity in very small angle of beach slope.



Note: **Curveliner kinetic motion with cetrifugive force at circular-rate center.**

$$ds = r d\theta \rightarrow d\theta / ds = 1/r. \quad \rho \mathbf{v} \cdot \text{grad} \mathbf{v} = \mathbf{f} = \rho v^2 / r = 2E_k / r.$$



**Appemdex\_4:Non linear solution by segment linearlization approximation.**

**(1)Simple example.**

At first,as for **linear(partial)differential equation**,solution is possible by anyhow.Then any non-linear equation can be expanded to linear segments as for the small variation formulation. Taking small variation of "f" yield following linear equation in general.

$$0 = G(f(t), f'(t), f''(t), \dots, f^{(n)}(t) \equiv df^n/dt^n).$$

$$0 = \delta f(t) \cdot \partial G / \partial f + \delta f'(t) \cdot \partial G / \partial f' + \dots + \delta f^{(n)}(t) \cdot \partial G / \partial f^{(n)}.$$

Solution in small interval  $[t_0 \leq t \leq t_0 + \Delta t]$ .  $\rightarrow \delta f = \delta f(t)$  ; with initial condition  $f(t_0), f'(t_0), \dots, f^{(n)}(t_0)$  are as follows.

**Next step are**

$$f(t_0 + \Delta t) = f(t_0) + \delta f(t_0 + \Delta t).$$

$$f(t_0 + 2\Delta t) = f(t_0 + \Delta t) + \delta f(t_0 + 2\Delta t).$$

.....

$$f(t_0 + N\Delta t) = f(t_0 + (N-1)\Delta t) + \delta f(t_0 + N\Delta t).$$

Thus solution  $f(t_0 \leq t \leq t_0 + N\Delta t)$  could be derived by small segments  $\{\delta f(t_0 + k\Delta t)\}$

**(2)Segment Linearlization of non-linear NS eqn by small variation expression.**

$$*D(\rho \mathbf{V})/Dt \equiv [\rho(t+dt, \mathbf{x}(t+dt)) \mathbf{V}(t+dt, \mathbf{x}(t+dt)) - \rho(t, \mathbf{x}(t)) \mathbf{V}(t, \mathbf{x}(t))] / dt.$$

$$= [\partial \rho / \partial t + \langle \mathbf{V}, \text{grad } \rho \rangle] \mathbf{V} + \rho [\partial \mathbf{V} / \partial t + \langle \mathbf{V}, \text{grad } \mathbf{V} \rangle].$$

$$*D(\rho \mathbf{V})/Dt = \mu \nabla^2 \mathbf{V} - \text{grad } P - 2\rho \boldsymbol{\Omega} \times \mathbf{V} + \rho \mathbf{g}.$$

$$\delta [D(\rho \mathbf{V})/Dt] =$$

$$[\partial \delta \rho / \partial t + \langle \delta \mathbf{V}, \text{grad } \rho \rangle + \langle \mathbf{V}, \text{grad } \delta \rho \rangle] \mathbf{V}$$

$$+ [\partial \rho / \partial t + \langle \mathbf{V}, \text{grad } \rho \rangle] \delta \mathbf{V} + \rho [\partial \delta \mathbf{V} / \partial t + \langle \delta \mathbf{V}, \text{grad } \mathbf{V} \rangle + \langle \mathbf{V}, \text{grad } \delta \mathbf{V} \rangle]$$

$$+ \delta \rho [\partial \mathbf{V} / \partial t + \langle \mathbf{V}, \text{grad } \mathbf{V} \rangle] = \mu \nabla^2 \delta \mathbf{V} - 2\rho \boldsymbol{\Omega} \times \delta \mathbf{V} - \text{grad } \delta P + \delta \rho \mathbf{g}.$$

$$\rho [\partial \delta \mathbf{V} / \partial t + \langle \mathbf{V}, \text{grad } \delta \mathbf{V} \rangle] = + \mu \nabla^2 \delta \mathbf{V}$$

$$- \rho \langle \delta \mathbf{V}, \text{grad } \mathbf{V} \rangle - [\partial \rho / \partial t + \langle \mathbf{V}, \text{grad } \rho \rangle] \delta \mathbf{V} - 2\rho \boldsymbol{\Omega} \times \delta \mathbf{V} - \langle \delta \mathbf{V}, \text{grad } \rho \rangle \mathbf{V}$$

$$- \delta \rho [\partial \mathbf{V} / \partial t + \langle \mathbf{V}, \text{grad } \mathbf{V} \rangle] - [\partial \delta \rho / \partial t + \langle \mathbf{V}, \text{grad } \delta \rho \rangle] \mathbf{V} - \text{grad } \delta P + \delta \rho \mathbf{g}.$$

Thus **NS eqn** could be **segment linearlized**.Also linear partial equations had been solved by approximation of time and space segmentization into linear algebraic simuletaneous equations.This is told **algorithm** for solving differential equation by software tools.

## Appendix\_5:Le Chaterier's Principle.

Thermodynamics is due to only two most reliable principles of 1st and 2nd law. Once the violation had occurred, someone could make free energy machine, but none could do.

This is great general conclusion of the thermodynamics which states that,

***If a system is in stable state, then any spontaneous change in its parameters must bring about process which tend to restore the system equilibrium.***

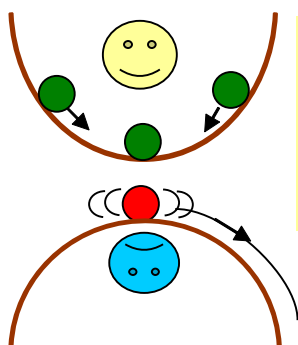
L.E Reichl, A modern Course in Statistical Physics, 1987, Texas.

For example, input heat on equilibrium state earth (1750), then what had happened?. The system must respond to decrease input influence. For increasing output heat from earth,

\* temperature rise to enhance cooling radiation to space (an example **strong draughts**).

\* hot ocean causes strong evaporation which turn to **strong storm with strong rain**.

\* **the deep meaning of "stability" in (thermo)dynamics.**



Note upper fig means stability of ball dynamics in bottom position where restoring force would occur, while lower fig means instability of ball dynamics, which tend to cause downward acceleration by small fluctuational force at the top position.

\* \* **Political health needs overwhelm restoring democracy movement.**

A force to restore healthy political state is decisive. Then people must recognize what is healthy (the stability point at bottom one (good and naive the lowest), but not top one (wicked the highest)).