Quantizable Gravity Field as Gauge one in Linear Coordinates

Quantum Gravity Dynamics(1)

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(Recived)

A gravity field become qunatizable in linear coordinates by localized Lorentz invariance as representaion of the principle of equivalence,which is simuletaneously and completely pure general gauge invariance without any exceptinal transform rule. Consequently the method gave Lagragian of qunatizable gravity field,which unifies all the interactions(Quantum Gravity Dynamics＝QGD).Then the phase transiton sequence became evident. SO(N＝11;1)⊃SO(11)⊃SO(10)⊃SU(5)⊃SU(3)×SU(2)×U(1).

1. Introduction.

Almost of interactions could be unified by general gauge field theory(the standard one). Then, is gravity field an exceptional one ?. As the fact, the foundation had already been completed in 1956 by the work of R.Utiyama[1,23)] in which the principle of equivalence(PE) on gravity force and force in accelated coordinates was represented by **localized Lorentz invariance**(it's a logical negation of **gloabal Lorentz invariance** in non-accelated coordinates). Then the invariance can become completely gauge one in linear coordinates, but not in curve linear ones. As the consequence, the quantum gravity field Lagragean(also the supreme unified field one) can instantly be derived as the general quantum gauage field theory[4,5,6)].

2. The theory of general relativity has two fatal defects.

Everyone has been considering necessity of *″curve linear coordinates″* for representing gravity field due to the general theory of relativity(GTR). However GTR has two fatal defects.

(F1) it becomes nothing definition for spinor's transformation.

(F2) The fundamental equation of GTR $G_{\mu\nu}=\kappa T_{\mu\nu}$ is illegal, because the left side $G_{\mu\nu}$ distortion tensor is pure one, while the right side $T_{\mu\nu}$ of momentum tensor is pseudo one.

3. A quantum gravity field is entirely a kind of gauge one.

Consequently GTR was abandoned in this theory, but employ the Principle of Equivalence(PE) for representing gravity field as ″localized Lorentz invariance″ due to the theory[1)].

A gravity field is equivalent to that in accelated coordinates.

Then in such coordinates,we could establish Localized Lorentz Transform(LLT) in each spot of time and space where localized uniform velocity of motion be observable.Then,especially note a fact that nothing necessity of employing curve-liniear coordinate for representing LLT.GTR was a cobody with curve-linear coordinate,but PE is free from it.Now let take LLT for spinor field ϕ and gauge one $A^{\alpha\beta}{}_{\mu}$.Our aim is to verify the infinitesimal transform formula of gauge field $A^{\alpha\beta}{}_{\mu}$ as that of pure gauge one in the invariace of gravity field Lagrangian with $\{\phi, A^{\alpha\beta}{}_{\mu}\}$.We denote linear coordinate $x \equiv (ict, x_1, x_2, x_3, (,,,, x_N))$. "i" in "ict" is imaginary number.Generalizing dimension(1 + 3) into (1+N)is entirely analogous in the invariant condition $dx^2 = dx^{\mu} dx_{\mu}$.Then,don't care on upper or lower suffix of covariant tensor and anti-covariant one such as $dx_{\mu} dx_{\mu} = dx^{\mu} dx_{\mu}$.It is the old fasion notation and you will see it essentiall in gaugeon gravity field in the later.Infinitesimal LLT are as follows.

$$dx'^{\alpha} \equiv (a^{\alpha}{}_{\beta}) dx^{\beta} = (\delta^{\alpha}{}_{\beta} + \varepsilon^{\alpha}{}_{\beta}(x)) dx^{\beta}. \tag{1}$$

$$\varepsilon^{\alpha}{}_{\beta}(x) = -\varepsilon^{\beta}{}_{\alpha}(x). \tag{2}$$

$$\partial / \partial x'^{\alpha} \equiv (\partial x^{\beta} / \partial x'^{\alpha}) \partial / \partial x^{\beta} = (a^{-1})^{\beta}{}_{\alpha} \partial / \partial x^{\beta}. \tag{3}$$

$$\phi_{\alpha}'(x') \equiv T_{\alpha}{}^{\beta} \phi_{\beta}(x) = [\delta_{\alpha}{}^{\beta} + ½ \varepsilon^{\sigma\tau}(x) G_{\sigma\tau\alpha}{}^{\beta}] \phi_{\beta}(x). \tag{4}$$

$$\gamma^{\sigma}\gamma^{\tau} + \gamma^{\tau}\gamma^{\sigma} = 2\delta^{\sigma\tau}. \ \langle \sigma, \tau = 0,1,2,3 \rangle. \tag{5}$$

$$G_{\sigma\tau} = ¼[\gamma^{\sigma}, \gamma^{\tau}] = ¼(\gamma^{\sigma}\gamma^{\tau} - \gamma^{\tau}\gamma^{\sigma}). \tag{6}$$

$$[G_{\eta\theta}, G_{\sigma\tau}] = f_{\eta\theta}{}^{\alpha\beta}{}_{\sigma\tau} G_{\alpha\beta}. \tag{7}$$

$a^{\alpha}{}_{\beta}$:LLT matrix for $\{dx^{\beta}\}$. $\langle a^{\alpha}{}_{\beta} a^{-1\beta}{}_{\alpha} = 1\rangle$

$\varepsilon^{\alpha}{}_{\beta}(x)$:arbitary infinitesimal space variable function.

$T_{\alpha}{}^{\beta}(x)$:LLT matrix for $\{\phi_{\beta}(x)\}$.

$G_{\sigma\tau}$:generator the speciall Lorentz Lie groupe.

γ^{σ}:Dirac gamma matrix.

$\overline{\phi} \equiv \phi^{*t}\gamma^{0}$:transposed and conjugated ϕ with γ^{0}.

$f_{\eta\theta}{}^{\alpha\beta}{}_{\sigma\tau}$:bonding constant in the Lie algebra SO(N;1).

Now we find infinitesimal transform of $A^{\alpha\beta}{}_{\mu}$ by assuming invariance of the Lagrangean density $\mathscr{L}(\phi, \partial_{\nu}\phi; A^{\alpha\beta}{}_{\mu}, \partial_{\nu}A^{\alpha\beta}{}_{\mu})$. Then note that $\overline{\phi}' = \overline{\phi}\, T^{-1}$, and $T^{-1}\gamma^{\mu} a^{-1\nu}{}_{\mu} T = \gamma^{\nu}$ are used.

$$\mathscr{L}'(x') \equiv -c\overline{\phi}(x)[\hbar\gamma^{\mu}(\partial'_{\mu} - \tfrac{1}{2}A'^{\alpha\beta}{}_{\mu}(x')G_{\alpha\beta}) + mc]\phi'(x')$$

$$= -c\overline{\phi}\, T^{-1}[\hbar\gamma^{\mu}(a^{-1\nu}{}_{\mu}\partial_{\nu} - \tfrac{1}{2}A'^{\alpha\beta}{}_{\mu}G_{\alpha\beta}) + mc]T\phi(x)$$

$$= -c\overline{\phi}\,[T^{-1}\hbar\gamma^{\mu}a^{-1\nu}{}_{\mu}\partial_{\nu} - \tfrac{1}{2}T^{-1}A'^{\alpha\beta}{}_{\mu}G_{\alpha\beta}) + mcT^{-1}]T\phi(x)$$

$$= -c\overline{\phi}\,[T^{-1}\hbar\gamma^{\mu}a^{-1\nu}{}_{\mu}\partial_{\nu}(T\phi) - \tfrac{1}{2}T^{-1}A'^{\alpha\beta}{}_{\mu}G_{\alpha\beta}T\phi + mc\phi(x)]$$

$$= -c\overline{\phi}\,[T^{-1}\hbar\gamma^{\mu}a^{-1\nu}{}_{\mu}T\partial_{\nu} - \tfrac{1}{2}\hbar\gamma^{\mu}A^{\alpha\beta}{}_{\mu}G_{\alpha\beta} + mc]\phi$$

$$- c\overline{\phi}\,[T^{-1}\hbar\gamma^{\mu}a^{-1\nu}{}_{\mu}\partial_{\nu}T \times + \tfrac{1}{2}\hbar\gamma^{\mu}A^{\alpha\beta}{}_{\mu}G_{\alpha\beta}$$

$$- \tfrac{1}{2}\hbar T^{-1}\gamma^{\mu}A'^{\alpha\beta}{}_{\mu}G_{\alpha\beta}T]\phi$$

$$= \mathscr{L}(x) - c\hbar\overline{\phi}\,[(T^{-1}\gamma^{\mu} a^{-1\nu}{}_{\mu} T) T^{-1}\partial_{\nu} T \times + ½\gamma^{\mu} A^{\alpha\beta}{}_{\mu} G_{\alpha\beta}$$

$$- ½ T^{-1}\gamma^{\mu} A'^{\alpha\beta}{}_{\mu} G_{\alpha\beta} T]\phi$$

$$= \mathscr{L}(x) - c\hbar\overline{\phi}\,[\gamma^{\nu} T^{-1}\partial_{\nu} T + ½\gamma^{\mu} A^{\alpha\beta}{}_{\mu} G_{\alpha\beta}$$

$$- ½ T^{-1}\gamma^{\mu} A'^{\alpha\beta}{}_{\mu} G_{\alpha\beta} T]\phi\,.$$

$$½\gamma^{\mu} A'^{\alpha\beta}{}_{\mu} G_{\alpha\beta} = ½ T\,\gamma^{\mu} A^{\alpha\beta}{}_{\mu} G_{\alpha\beta} T^{-1} - T\,\gamma^{\nu}\partial_{\nu} T^{-1}. \qquad (8)$$

The conclusion of pure gauge transform property of $\{A^{\alpha}{}_{\mu}\}$ is $\delta A^{\alpha} = \{\partial_{\mu}\varepsilon^{\alpha} + \varepsilon^{\sigma} A^{\beta}{}_{\mu} f_{\sigma}{}^{\alpha}{}_{\beta}\}$. Now let's prove it.
Then we use following co-variant derivative relations.

$$\partial_{\mu}\varepsilon^{\alpha}(x) = A^{\alpha}(x)_{\mu}. \qquad (9)$$

$$D_{\mu}\phi_{\alpha}(x) \equiv \lim\ \Delta x_{\mu} \to 0,$$

$$[\phi_{\alpha}(x^{\mu}+\Delta x^{\mu}) - \{\phi_{\alpha}(x^{\mu}) + \varepsilon^{\theta}(x^{\mu})\,\mathbf{Q}_{\theta\alpha}{}^{\beta}\phi_{\beta}(x^{\mu})\}]/\Delta x_{\mu}$$

$$= \partial_{\mu}\phi_{\alpha} - A^{\theta}{}_{\mu}\mathbf{Q}_{\theta\alpha}{}^{\beta}\phi_{\beta}. \qquad (10)$$

$$T = [1 - ½\,\varepsilon^{\sigma\tau}(x)\,G_{\sigma\tau}]、\ T^{-1} = [1 + ½\,\varepsilon^{\sigma\tau}(x)\,G_{\sigma\tau}]. \qquad (11, 12)$$

$$0 = \partial_{\mu}(\varepsilon^{\sigma\tau}\varepsilon^{\alpha\beta}) = A^{\sigma\tau}{}_{\mu}\varepsilon^{\alpha\beta} + \varepsilon^{\sigma\tau} A^{\alpha\beta}{}_{\mu}. \qquad (13)$$

$$\tfrac{1}{2}\gamma^{\mu}\delta A^{\eta\theta}{}_{\mu}G_{\eta\theta} \equiv \tfrac{1}{2}\gamma^{\mu}[A'^{\alpha\beta}{}_{\mu} - A^{\alpha\beta}{}_{\mu}]G_{\alpha\beta}$$

$$= \tfrac{1}{2}[1 - \tfrac{1}{2}\varepsilon^{\sigma\tau}G_{\sigma\tau}]\gamma^{\mu}A^{\alpha\beta}{}_{\mu}G_{\alpha\beta}[1 + \tfrac{1}{2}\varepsilon^{\sigma\tau}G_{\sigma\tau}]$$

$$-[1 - \tfrac{1}{2}\varepsilon^{\sigma\tau}G_{\sigma\tau}]\gamma^{\mu}\partial_{\mu}[1 + \tfrac{1}{2}\varepsilon^{\alpha\beta}G_{\alpha\beta}] - \tfrac{1}{2}\gamma^{\mu}A^{\alpha\beta}{}_{\mu}G_{\alpha\beta}$$

$$= -\tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}G_{\sigma\tau}\gamma^{\mu}G_{\alpha\beta} + \tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}\gamma^{\mu}G_{\alpha\beta}G_{\sigma\tau}$$

$$-\tfrac{1}{2}\partial_{\mu}\varepsilon^{\sigma\tau}\gamma^{\mu}G_{\sigma\tau} + \tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}G_{\sigma\tau}\gamma^{\mu}G_{\alpha\beta}$$

$$= +\tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}\gamma^{\mu}G_{\alpha\beta}G_{\sigma\tau} - \tfrac{1}{2}\partial_{\mu}\varepsilon^{\sigma\tau}\gamma^{\mu}G_{\sigma\tau}$$

$$= -\tfrac{1}{4}A^{\sigma\tau}\varepsilon^{\alpha\beta}{}_{\mu}\gamma^{\mu}G_{\sigma\tau}G_{\alpha\beta} - \tfrac{1}{2}\partial_{\mu}\varepsilon^{\sigma\tau}\gamma^{\mu}G_{\sigma\tau}$$

$$= -\tfrac{1}{2}\gamma^{\mu}\{\partial_{\mu}\varepsilon^{\eta\theta}G_{\eta\theta} + \tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}[G_{\sigma\tau}、G_{\alpha\beta}]\}$$

$$= -\tfrac{1}{2}\gamma^{\mu}\{\partial_{\mu}\varepsilon^{\eta\theta} + \tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}f_{\sigma\tau}{}^{\eta\theta}{}_{\alpha\beta}\}G_{\eta\theta}.$$

$$\delta A^{\eta\theta} = -\{\partial_{\mu}\varepsilon^{\eta\theta} + \tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}f_{\sigma\tau}{}^{\eta\theta}{}_{\alpha\beta}\}.$$

Then turning sign $\varepsilon^{\eta\theta} \to -\varepsilon^{\eta\theta}$ with remebering (11,12).

$$\delta A^{\eta\theta} = \{\partial_{\mu}\varepsilon^{\eta\theta} + \tfrac{1}{4}\varepsilon^{\sigma\tau}A^{\alpha\beta}{}_{\mu}f_{\sigma\tau}{}^{\eta\theta}{}_{\alpha\beta}\}. \qquad (14)$$

Remeber pure gauge transform of $\delta A^{\alpha} = \{\partial_{\mu}\varepsilon^{\alpha} + \varepsilon^{\sigma}A^{\beta}{}_{\mu}f_{\sigma}{}^{\alpha}{}_{\beta}\}$. Consequnetly gravity field was found to be pure gauge field, which is quantizable as rules of the established general gauge field as the standard theory.

－REFERRENCES－

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APPENDIX1:The formulation of general guage field Lagrangian.

$$\mathscr{L}_{QGD}=-c\overline{\phi}\,[\hbar\gamma^{\mu}(\partial_{\mu}+gA^{a}{}_{\mu}\mathbf{Q}_{a})]\phi+icB^{a}\partial_{\mu}A^{a}{}_{\mu}+\tfrac{1}{2}\alpha B^{a}B^{a}$$
$$-(1/4\eta)(\partial_{\mu}A^{a}{}_{\nu}-\partial_{\nu}A^{a}{}_{\mu}+gf_{b}{}^{a}{}_{c}A^{b}{}_{\mu}A^{c}{}_{\nu})^{2}$$
$$+\chi\overline{C}^{a}\cdot\partial_{\mu}(\partial_{\mu}C^{a}+f_{b}{}^{a}{}_{c}A^{b}{}_{\mu}C^{c}).$$

－note－

*single suffix "a" is an abribation of double suffix($\alpha\beta$),where $\alpha\neq\beta\equiv0,1,2,3,....,N$ in (N+1) dimensional suffixes in QGD.

*time and space variable x of (1+11) dimension and the infinitesimal Lorentz transform,inifinitesimal gauge transform.

$x_{\mu}\equiv(x_{0}\equiv ict,x_{1},x_{2},x_{3},...,x_{11})$.Note using "imaginary number ict".

$dx_{\alpha}\equiv(\delta_{\alpha}{}^{\beta}+\varepsilon_{\alpha}{}^{\beta})dx_{\beta}$.<<infinitesimal Lorentz transform for space variable>>

$\delta A^{a}{}_{\mu}=\partial_{\mu}\varepsilon^{a}+\varepsilon^{b}f_{b}{}^{a}{}_{c}A^{c}{}_{\mu}$.<<guage field tranform with infinitesimal$\{\varepsilon^{a}(x)\}$>>.

$\partial_{\mu}\varepsilon^{a}=A^{a}{}_{\mu}$.

This is the essential formulation as for the QGD theory enabling imaginary(aniti-herimitian) and transversal gauge field component $A^{a}{}_{k}$ which realize negative energy in the initial creation of this universe.

*filed variables:

{ϕ = spinor field, B^a = canonical conjugate field of $A^a{}_0$,

C^a = FP gohst}.

*physical constant:

{c = light velocity, $\hbar$ = Plank, η = permibility, χ = FP gohst

$\alpha = -1/\varepsilon$ (Feyman gauge constant = -1/(permittivity in QED)}.

APPENDIX2:Lorentz covariance of Dirac equation:

$[\hbar\gamma^{\alpha}\partial'_{\alpha}+mc]\phi'(x')=[\hbar\gamma^{\alpha}(a^{-1})^{\beta}{}_{\alpha}\partial/\partial x^{\beta}+mc]T\phi(x)$

$=T[T^{-1}\hbar\gamma^{\alpha}(a^{-1})^{\beta}{}_{\alpha}\partial/\partial x^{\beta}+mcT^{-1}]T\phi$

$=T[\hbar(T^{-1}\gamma^{\alpha}(a^{-1})^{\beta}{}_{\alpha}T)\partial/\partial x^{\beta}+mc]\phi=T[\hbar\gamma^{\beta}\partial_{\beta}+mc]\phi(x).$

$\rightarrow\ T^{-1}\gamma^{\mu}a^{-1\nu}{}_{\mu}T=\gamma^{\nu}.$

$\overline{\phi}'(x)\equiv(T\phi(x))^{*t}\gamma^{0}=(\phi^{*t}(x)T^{*}\gamma^{0}=\phi^{*t}(x)\gamma^{0}T^{-1}=\overline{\phi}(x)T^{-1}.$

$(T^{*}\gamma^{0})T=(\gamma^{0}T^{-1})T=\gamma^{0}.\ \rightarrow T^{*}\gamma^{0}=\gamma^{0}T^{-1}.$