Gödel Incompleteness Theorem and Statistical Phenomena (Pan Statistical Theorem)'2008/01/03,2016/01/03(English Version)

It is very curious that actuality of Gödel incompleteness theorem has been told nothing at all !!.

①The fact is statisticalization due to information loss in cause.

②The theorem is due to **indeterminacy of infinity**$=\infty$ **in Natural Number Theory.**

③Real number zero 0(R)$=1/\infty$ is indefinite,but finite 0(R)=0(N)<natural number 0>.

This is a **contradiction realizing** in real number zero=0(R).but harmless by non observable.

④Thereby **probability 0(R)** is contradictory. A sample process of stochastic one has zero probability,which causes local observability,while non-observable in global<fluid chaos>.

[1] : Göedel Incompleteness Theorem and Contradiction Realizing of Real Number 0.

❶Göedel Incompleteness Theorem:

If theory K with natural number one is non contradictory,a proposition in K is undetermined.

Proof)The maximum number of natural number set $N\equiv\{0, 1, 2, 3, \ldots\ldots\}$is undetermined. If M was proved maximum,then(M+1)$>$M and (M+1) is element of N,which is contradiction.Thus our comprehensibility may belongs to **finiteness**.

❷Theorem of Contradictory of Real Number "0" :

Rational number set $R\equiv\{1,1/2,1/3,\ldots,1/n,..\}$'s minimum value Z is undetermined.

proof)Z must be combined with uncertain M in above ❶ by one to one in Z=1/M.

However real number 0 **(R)=natural number** 0 **(N)** is determined.

As for $0<\forall\delta, n>\exists n_0=1+\text{intger}[1/\delta]$, following in-equation is established.

$$|1/n-0|<|1/n_0-0|=|1/(1+\text{intger}[1/\delta])-0|<|1/[1/\delta]-0|=\delta.$$

Thus minimum value of $R\equiv Z=$**real number** 0 **'s** simultaneous **definiteness** and **indefiniteness** are proved **contradictory.**However this contradictory is not harm,but benefit.

❸Realizing Contradiction and Theory Destruction Theorem(review).

"Realizing Contradiction<(**A**∩¬ **A**) can make any proposition **B** true".

proof)$\neg A\subset(A\subset B)\equiv C\equiv 1.\ldots(1)$

conditional : $A\subset B$ is always 1,exception is $A=1, B=0\rightarrow(A\subset B)=0$、

Thereby if $\neg A=0$, then $C=1$,and if $\neg A=1$, then $(A\subset B)=1$, **so** $C\equiv1$.

Thus we can prove (1) is tautology.

(2)Due to assumption $\neg A=1$, so $(A\subset B)=1$。

(3)Due to assumption $A=1$. so $(A\subset B)=1$。 Thus arbitrary B=1 is proved。

❹Real number “0” probability from View of Quantum Theory.

⑴**Real number 0(probability) means non observable,but not non-being !!!.**

The most famous,but the most curious fact is that *elementary particle size is zero* in **Quantum Filed Theory(QFT)** the established standard elementary particle one.

An elementary particle can not have finite size due to upper limit of light velocity $= c_0$ *in special theory of relativity.If finite,which must be rigid body,and signal can transfer with infinitive velocity.*

. . . . Landau & Lifshitz,Classical Theory of Field,Moscow,1962,(Tokyo Tosho,1970)

Only the standard theory can agree with experimental facts.None can succeed finite size elementary particle theory!!!.

⑵Nothing harm,but benefit by real number o’s contradictory

Real number 0 is **indefinite** due to $0 = 1/\infty$,while it is simultaneously **finite** due to real number 0 ＝natural number 0.This contradiction never be observable,so harmless.On the contrary,it could save interpretation difficulty of size 0 elementary particle.Also see APPENDIX-4.

⑶Origin of Probability＝information loss due to incompleteness.

A mathematical singularity is to be information loss<incompleteness>.

In fact, the probabilistic phenomena is actual in experiments !!!.

example-1) **Hyper Function**(the definition by example of Dirac’s delta function).

$\delta(x) \equiv \lim_{\varepsilon \to +0} (1/2\pi i)[1/(x - i\varepsilon) - 1/(x + i\varepsilon)]$. ....x=0 is singular point in ε=0.

Above function can be regular so long as $\varepsilon \neq 0$.<*lim* operation is taken at last !>

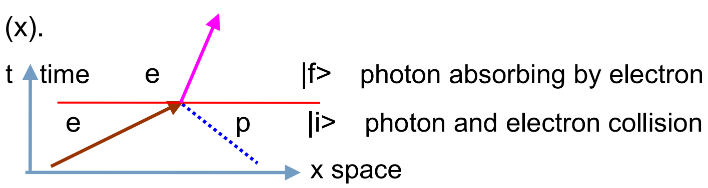
Filed operator in QFT are **hyper function** in general such as Dirac’s delta one.

$\{\phi_\alpha(x_0,\mathbf{x})\phi^+_\beta(x_0,\mathbf{y}) + \phi^+_\beta(x_0,\mathbf{y})\phi_\alpha(x_0,\mathbf{x})\} = i\hbar\delta_{\alpha\beta}\delta(\mathbf{x} - \mathbf{y})$.

Commutation relation{..} of field operator is defined by delta function.Then **product of hyper function** in same singular point (x)never be mathematically defined<origin of information loss>.Interaction Hamiltonian in Quantum Electro Dynamics(QED) is expressed by field operator(same singular point x) product as follows.

$H_{QED}(x) = e\bar{\phi}(x)\gamma_\mu A_\mu(x)\phi(x)$.

$p_{fi} = \langle f|H_{QED}(x)|i\rangle$



p_{fi} is **probability amplitude** of QED reaction from initial state=|i> to final state=|f> such as electron= $\phi(x)$ and photon= $A_\mu(x)$ one.Thereby such $H_{QED}(x)$ can not be mathematically regular,however which is lucky to yield probability amplitude of reactions.

＊reference:P9,87,N.Nakanishi,**Quantum Field Theory**(Japanese),Baihuhkan,1975,Tokyo.

example-2) **Canonical Distribution by Lagrange's undetermined coefficient method.**

$E = \Sigma_i p_i e_i$. known average value

$S = k\Sigma_i p_i(1/\ln p_i)$. entropy for information measure(**the non biased estimation**).

$0 = \partial/\partial p_i\{k\Sigma_i p_i(1/\ln p_i) + \Sigma_i p_i e_i\} = k(1/\ln p_i) - k + e_i = -k\ln p_i - k + e_i$

$\ln p_i = -1 + e_i/k$. $\rightarrow$ $p_i = \exp(e_i/k - 1) = N\exp(\beta e_i)$.

Also Canonical Distribution is confirmed in experiment in statistical dynamics.
Non Biased Estimation is due to **maximizing entropy**(least information measure)
in **constrained condition** such as average value,Unless ordered flow input and output,
a nature in **closed system** tends to be max chaos.

example-3) **Time independent transition probability ! .**

Following are history evolution mechanism told by Quantum Field Theory View.
Time evolution in this world could be told as **vanishing past state**=|initial> and **creating coming future state**=|final> by something **dynamics**$=H_{QFT}=k(P_f\leftarrow P_i)$**F*P**. $\langle$final$|H_{QFT}|$initial$\rangle$ $=\langle f|k(P_f\leftarrow P_i)$**F*P**$|i\rangle$=**transition probability** amplitude from |i> to |f>. **F*P** is operator vanishing past and creating future with **something legacy conserved**.$k(P_f\leftarrow P_i)$ is something function dominating probability due to past and future **state variables change**$(P_f\leftarrow P_i)$.That is, time evolution is composed from **something conserved** and **something changed.** Then calculated probability is determined **unique** due to something **dynamics**$=H_{QFT}$. Such probability can be **time independent value** as the principle<this is very,very coarse explanation ?!>.

example-4) **Stochastic Process<time dependent transition probability> ! .**

Quantum Process(time dependent)in general is called stochastic process dominated by transition probability P_{fi}(3).Possible states in a physical systems are{ |0>,|1>,....|s>,...}.
The probability of state $|s\rangle$ at time $t=\omega_s(t)$. $\Gamma_{su}(t)$=time dependent state transition probability (u→s)in unit time.

$d\omega_s(t)/dt = \Sigma_u \Gamma_{su}(t)\,\omega_u(t) - \Sigma_s \Gamma_{us}(t)\,\omega_s(t)$.............. **Master Equation**.

state change rate={inflow transition −outflow transition} as budget account.

Telling as for the conclusion for quantum stochastic process,

$\Gamma_{su}(t) = P_{su}/\Delta t(t) = (\Delta E(t)/\hbar)T_{su}$. $\{P_{su} \equiv |\langle s|\mathbf{H}|u\rangle|^2\}$.

$\Delta t(t)\,\Delta E(t) = \hbar$. Time and energy uncertainty **theorem** in **statistical ensemble**.

$\Delta E(t) = \{\Sigma_s \omega_s(t) \langle E_s - \langle E\rangle\rangle^2\}^{1/2}$. Energy deviation in **statistical ensemble**.

http://www.777true.net/img007-Quick-Guide-to-Quantum-Stochastic-Mechanics.pdf

［2］：Gödel Incompleteness Theorem belongs to Statistical Phenomena.

(Pan Large Number Law、Pan Statistic Theorem)：

If **conditional proposition** X＝A(cause)⊂B(result) is incomplete,or also X＝A(cause)⊂¬B(result). This is condition of **non-contradictory** and **indeterminism** in result in X.Then the **indeterminism** is to assume being of **indefinite trial** for X. Because the result must be always **random** not to be unique.n times trial in X's results is denoted as β_n＝B, or β_n＝¬ B.

Of course the result must be **observable**.The observed series are $\beta \equiv \{\beta_1, \beta_2, \beta_3, ..., \beta_n, ...\}$, which are **random results series**.Then we could prove to define **experienced probability value** and the converging theorem in different way from usual one.

－Pan Large Number Law、Pan Statistical Theorem－

2 observable elements phenomenon(event)≡{e, ¬ e},which are exclusive with each other in each indefinite observing trial.

S1:Observed event is unique in each trial<non contradictory observability>

S2:Observing trial times can be indefinite<possibility of indefinite trial>.

In N times trial,observing times $e = n_e(N)$, that of $\neg e = n_{\neg e}(N)$、then by $N \to \infty$、$P_e(N) \equiv n_e(N)/N \to \alpha$、$P_{\neg e}(N) \equiv n_{\neg e}(N)/N \to \beta$ are converged to be $1 \geqq \exists \alpha$、$\exists \beta \geqq 0$、$\alpha + \beta = 1$. That is,

$$\{\forall \varepsilon > 0、\forall M > 0 ;\ ; \exists N_0,\ M + N_0 \geqq \forall N \geqq N_0 ;\ ; 1 \geqq \exists \alpha、\exists \beta \geqq 0、\alpha + \beta = 1\}$$
$$\to |n_e(N)/N - \alpha| = |n_{\neg e}(N)/N - \beta| < \varepsilon .$$

proof)$N \equiv N_0 + \Delta N$, $n_e(N) \equiv n_e(N_0) + \Delta n$, $\alpha \equiv n_e(N_0)/N_0 \equiv n_0/N_0$、Those 3 equations are defined. Note $n_e(N) + n_{\neg e}(N) = N$、$|\Delta n - \Delta N \alpha| < \Delta N$(in proof process),

$$\Delta \equiv |n_e(N)/N - \alpha| = |n_{\neg e}(N)/N - \beta| = |(n_0 + \Delta n)/(N_0 + \Delta N) - n_0/N_0|$$
$$= |(n_0/N_0 + \Delta n/N_0) - n_0/N_0 - (\Delta N/N_0)(n_0/N_0)| / (1 + \Delta N/N_0)$$
$$= |\Delta n/N_0 - (\Delta N/N_0)\alpha| / (1 + \Delta N/N_0) = |\Delta n - \Delta N \alpha| / N_0(1 + \Delta N/N_0)$$
$$\leqq \Delta N| / N_0(1 + \Delta N/N_0) \leqq \Delta N| / N_0 \leqq M| / N_0.$$

Thereby as for $\forall M$、$\forall \varepsilon$, taking N_0 as $M/\varepsilon < N_0$, then $\Delta < \varepsilon$. (proof end)

In above proof,you may feel curious for **finite interval of M** for **probability converging**. However **nothing upper limit of M**,so nothing problem.The modern probability theory by Kolmogrov never assign probability value itself.In our theory ,being a value was proved.

This is statistical-ization of incompleteness phenomena.

APPENDIX_1:The Weak Laws of Large Number.

Many “n times” trial establishes convergence of observed average value.In fact,statistical mechanical variables agree with this theorem.

(1)**Markov’s inequation**.

$\mu \equiv E[X] \equiv \int_0^\infty dx.xf(x) \geqq \int_a^\infty dx.xf(x) \geqq a\int_a^\infty dxf(x) = a\ P(X \geq a)$

$P\{X \geq a\} \leqq E[X]/a.$

(2)**Chebychev’s inequation.**

$P\{|X - \mu| \geq a\} = P\{|X - \mu|^2 \geqq a^2\} \leqq E[(X - \mu)^2]/a^2.$

(3)**The Weak Laws of Large Number**

$E[(S_n/n - \mu)^2] = nE[(X - \mu)^2] = n\sigma^2.$

$P(|S_n/n - \mu| > \varepsilon) = P(|S_n - n\mu|^2 > n^2\varepsilon^2) \leqq n\sigma^2/n^2\varepsilon^2 = \sigma^2/n\varepsilon^2.$

$\rightarrow \quad P(|S_n/n - \mu| > \varepsilon) \leqq \sigma^2/n\varepsilon^2. \rightarrow \quad P(|S_n/n - \mu| < \varepsilon) \geqq 1 - \sigma^2/n\varepsilon^2.$

“Many times trial tends to converge average value by probability＝1”.

＊$S_n \equiv X_1 + . . . + X_n.$

$\sigma^2 \equiv E[(X - \mu)^2].$

APPENDIX_2:Observed physical value formulation in QM<statistical theory>.

A observable physical value＝a (real number)is defined as eigen equation formulation in QM.

$\bar{A}\phi_a = a\phi_a.$

$\bar{A}$＝Hermite operator of physical variable $\bar{A}$.

ϕ_a＝eigen function with eigen value for operator＝$\bar{A}$.

If quantum state is ϕ、then physical value of $\bar{A}$ is ensemble average value＝<a>.

$\phi = \int da.K(a)\phi_a.$ expansion theorem by orthogonal eigen function series.

$\rightarrow \langle \phi_a | \phi \rangle = K(a).$ probability amplitude of eigen state ϕ_a in ϕ. $\langle \phi_a | \phi_b \rangle = \delta(b-a).$

$\langle \phi | \phi \rangle \equiv \int dx \phi^*(x) \phi(x).$ inner product of vectors in function space.

$\langle a \rangle = \langle \phi | \bar{A}\phi \rangle = \langle \int db.K(b)\phi_b | \bar{A} \int da.K(a)\phi_a \rangle = \langle \int db.K(b)\phi_b | \int da.aK(a)\phi_a \rangle$

$= \int db \int da.a.K(b)^*K(a)\delta(b-a) = \int da.a|K(a)|^2 \ldots$ $|K(a)|^2$ is probability density for “a”.

APPENDIX_3:Quantum Vacuum World is Anything Can Be<Almighty One!!>

Simply to tell,quantum vacuum can create matter of elementary particle and anti-particle pair of $\{a^+,a^-;g\}$**from nothing**.It is called “vacuum polarization” supported both by theory & experiment. It is entirely **contradict** ordinal **law “*nothing is nothing forever*”**.

A **realizing contradiction** can make anything true(＝realizing anything !!)<see **[1]** : ❸>.

∗Note physical value of $\{a^+,a^-\}$are ± symmetric to conserve total physical value as $0=a^+ + a^-$.
But exception is **energy** which is canceled by that of negative gravity field energy$=E^g$ as $0=E^g+2E(a^+ + a^-)$.An **attraction force**(gravity) has **negative energy** in general.

Thus quantum vacuum world is singular enough ,where anything can be simultaneously.
A being of probability seems anti-symmetry between a realizing world and a non realizing world.
However total universe(multi-lateral world)is almighty world<see “The being of parallel worlds”>.
http://www.777true.net/Logic-the-most-simple_but-supreme-way-for-recognition.pdf
So Einstein once told ***God never throw dice !!***.

Then if elementary particle has finite size,it would become impossible to pack infinitive pieces of elementary particles in finite space.A nature is great enough to allow us the comprehension.
http://www.777true.net/Proof-on-God.pdf.

APPENDIX_4:旧日本語版。

－Goedel不完全性定理の確率統計現象性と汎統計学定理－ 08/1/3:

Goedel不完全性定理ほど奇怪議論はない、その真相が語られない！！、

①その真相は一つは非決定性に起源する**情報喪失-確率化現象**という事である。

②その前に同不完全性定理が**自然数N**での非決定性＝**無限大**に起因する事、

③それが**実数０の非決定、決定の矛盾実現**と一対にある事、

④決定論にして局所確定、大局不確定の**カオス**は確率過程統計集団の決定論的存在である実現確率０の標本過程である事。

［１］：Goedel不完全性定理と実数０の矛盾実現性：

❶Goedel不完全性定理：

数論Nを含む任意の理論系Kが無矛盾ならば、命題Xが存在し、その真偽決定は不可能。

証明）自然数集合N≡{0, 1, 2, 3, ... } の最大値Mを考えるとMは決定不可能。

もしMが最大値と決定ならば（M＋1）＞Mで且つ、（M＋1）はNの要素だから矛盾明白。

この証明一つで「**有限の立場**」が完全（決定論）性に不可欠が判ります。

❷実数０の矛盾性定理：

有理数集合R≡{0, 1, 1/2, 1/3, . . . , 1/n, . . } の最小値Zを考えるとZは決定不可能。

証明）それはNの最大値MとZ＝1/Mで１対１対応せねばならないは明白。

しかもZ＝自然数０とも確定する。

０＜∀δに対して n＞$\exists n_0$＝1＋intger[1/δ]に関して以下不等式が成立。

|1/n－0|＜|1/n_0－0|＝|1/(1＋intger[1/δ])－0|＜|1/[1/δ]－0|＝δ.

「かようにR最小値Z＝**実数０**の不確定と同時に確定と言う**矛盾成立**が証明された」。

❸矛盾実現と定理系崩壊定理：

「命題A, 否定命題¬ Aが同時に真（矛盾実現）になると任意命題Bも真になる」。

証明）⑴¬ A⊂（A⊂B）≡C≡1.

条件法命題：A⊂BはA＝1, B＝0に限り（A⊂B）＝０、それ以外は全部 1。

故に¬ A＝０ならばC＝1, 更に¬ A＝１ならば（A⊂B）＝1 でC≡1.

かくて⑴式の恒真性が証明された.

⑵仮定に従い¬ A＝１だから、（A⊂B）＝1。

⑶仮定に従いA＝１で且つ、（A⊂B）＝1。だから**任意命題B＝1が証明された。**

❹確率値＝実数０だと**非可観測**とするのが量子物理学の立場だが、上記崩壊定理により、「矛盾性が発現」するがそれが「可観測であり、非可観測」の意味になる≡**カオス性**。
但し矛盾実現確率０で、**理論系整合性**に支障はないと筆者等は見る！。正確には
有限値確率無矛盾性と再定義すべきなのだろう。

❺従来のカオス性の定義との合致性：

決定論的アルゴリズムの下に解が局所的には可観測、大域的には非可観測が最大公約数！。

⑴本報告主張はカオスは確率過程の「**実現確率値＝０の標本過程**」。
⑵この定義は標本過程の決定論性から**決定論的アルゴリズム下の解に合致。**
　さいころの名目は決定論的統計標本で確率値は有限の 1/6 と言う次第。
⑶**実現確率値**＝「**実数０の矛盾性から局所可観測、大局非可観測**」の性質にも合致。
　「**但しどこまでが局所的で、大局的かの基準は何も述べない**」事に注意！。

☞：恐縮だが筆者はカオス論には殆ど無縁、サイト情報等によると，各種広域研究があり、「非線形性」と絡めた話もあるが，上記定義では決定論と言う大前提があり、まず方程式での特異点での**非因果遷移**を例外にせねば前提が崩れる事、非線形では自己自己相互作用での**因果的不安定性増大等での結果多様発散化**も想定されて、これまでカオスに入れる事には懸念？。

[２]：Goedel 不完全性定理の確率統計現象性(汎大数法則、汎統計学定理)：

上記[１]：Goedel 不完全性定理の問題となる命題Ｘを原因Ａの下に結果Ｂとその否定命題┓Ｂが発生するの条件法一般命題と見れば、**無矛盾性と非決定性**から、ある試行では結果 β_n＝Ｂ，又は┓Ｂの一つに確定するので**観測可能**。この**観測試行列：**
$\beta\equiv\{\beta_1, \beta_2, \beta_3, ..., \beta_n, ...\}$を考えるとランダム列でなければならず、しかも**Ｂ実現確率値が定義**され、且つ大数収束する事が一般証明できる。いささか従来の収束法則とは異質だが、異論が挟めない論理にできる**(汎統計学定理)**。

－汎大数法則、汎統計学定理－

無限反復観測可能な排他識別可能な２現象集合≡{e，┓e}があり、

S1:個々の観測毎に現象は一つに識別観測可能<可観測性>.

S2:この観測は望む限り反復観測可能<無限試行可能性>.

この時Ｎ回の観測でｅを見る頻度 $n_e(N)$,┓ｅのそれを $n_{┓e}(N)$とすればＮ→∞で、
$P_e(N)\equiv n_e(N)/N$、$P_{┓e}(N)\equiv n_{┓e}(N)/N$、は $1\geqq\exists\alpha$、$\exists\beta\geqq 0$、$\alpha+\beta=1$ に収束.即ち

$$\{\forall\varepsilon>0、\forall M>0\ ;\ ;\ \exists N_0,\ M+N_0\geqq\forall N\geqq N_0;\ ;\ 1\geqq\exists\alpha、\exists\beta\geqq 0、\alpha+\beta=1\}$$
$$\rightarrow|n_e(N)/N-\alpha|=|n_{┓e}(N)/N-\beta|<\varepsilon。$$

証明)$N\equiv N_0+\Delta N$、$n_e(N)\equiv n_e(N_0)+\Delta n$、$\alpha\equiv n_e(N_0)/N_0\equiv n_0/N_0$、以上３式を定義。
$n_e(N)+n_{┓e}(N)=N$ に留意、かつ計算途中で$|\Delta n-\Delta N\alpha|<\Delta N$ である事を考慮すれば

$$\Delta\equiv|n_e(N)/N-\alpha|=|n_{┓e}(N)/N-\beta|=|(n_0+\Delta n)/(N_0+\Delta N)-n_0/N_0|$$
$$=|(n_0/N_0+\Delta n/N_0)-n_0/N_0-(\Delta N/N_0)(n_0/N_0)|/(1+\Delta N/N_0)$$
$$=|\Delta n/N_0-(\Delta N/N_0)\alpha|/(1+\Delta N/N_0)=|\Delta n-\Delta N\alpha|/N_0(1+\Delta N/N_0)$$
$$\leqq\Delta N|/N_0(1+\Delta N/N_0)\leqq\Delta N|/N_0\leqq M|/N_0.$$

故に任意の M、任意のεに着き $M/\varepsilon<N_0$ なる整数 N_0 を取れば$\Delta<\varepsilon$。(証明終)

上記証明では確率値が収束する有限試行区間Ｍの存在に奇異を感じるだろうが、これは任意で**上限が存在しない**から問題無し。現代確率論を確立した Kolmogrov 流確率論では確率値その物は要素に天下りに与える。我々は値その物を与えないが**超大数法則**が成立する確率値存在を証明した(不完全命題の確率統計化)。