Information Loss Process in NS Equation＝Cause of The Chaos. 2013/10/3,22

Long term prediction is our imminent wish,while chaos in NS eqn solution has been intercepting.The cause of chaos is due to **frictional term** $\mu \nabla^2 \mathbf{V}$,which could be showed to become heat($J_i \equiv \mu [(\mathrm{div}\mathbf{V})^2 + (\mathrm{curl}\mathbf{V})^2]$),but not dynamical momentum energy.
This is also nothing,but entropy increasing process(dS＝dE/T) which could be a measure for **Chaos**(**the irreversibility**) in Navier-Stokes Fluid Equation.

[1] : Friction Force as Heat Dissipative Process.

The most simple example of heat dissipating process is dynamics of friction force.

$\mathbf{F} = d(m\mathbf{V})/dt = -k(\mathbf{V}/V)/V$.

＊**V** is velocity vector and **F** is a model of friction force proportional to (1/V).

$dE/dt = \langle \mathbf{V}.\mathbf{F} \rangle = d(m\mathbf{V}^2/2)/dt = -k < 0$.

$E(t_1) - E(t_0) = \int_{t0}^{t1} dt\,(dE/dt) = \int_{t0}^{t1} dt\, d(m\mathbf{V}^2/2)/dt = \int_{t0}^{t1} dt\,(-k) = -k(t_1 - t_0) < 0$.

That is,initial positive kinetic energy $(m\mathbf{V}(t_0)^2/2)$ is absorbed as a heat one to stop m**V**.

Also note **friction force work(energy)** indicates generating **negative energy**/time.

[2] : Energy Theorem in NS-Fluid Equation,

(1) $D(\rho \mathbf{V})/Dt = \mathbf{f} = \mu \nabla^2 \mathbf{V} - \mathrm{grad} P + 2\rho_s \mathbf{\Omega} \times \mathbf{V} + \rho \mathbf{g}$. <NS equation>

Acceleration force＝{frictional＋pressure＋Coriolis＋gravity}forces.

(2) **Energy Theorem** : $E(t) - E(0) = \int_0^t dW = \int_0^t dt\, \mathbf{V} \cdot \mathbf{f}$

$\int_0^t dt\, \mathbf{V} \cdot D(\rho \mathbf{V})/Dt = \int_0^t dt\, \mathbf{V} \cdot \mu \nabla^2 \mathbf{V} - \int_0^t dt\, \mathbf{V} \cdot \mathrm{grad} P + \int_0^t dt\, \mathbf{V} \cdot \rho \mathbf{g}$.

☞:note energy from **Coriolis force** is vanished due to $0 = \langle \mathbf{V} \cdot 2\rho_s \mathbf{\Omega} \times \mathbf{V} \rangle$.

If $0 = \langle \mathbf{V} \cdot \mathbf{g} \rangle$, also the energy from gravity becomes zero. Ordinal flow $\mathbf{V}$ in ocean and atmosphere is orthogonal to $\mathbf{g}$, so gravity contribution is **vertical flow** $\mathbf{V}$ only.

(a) $K \equiv \int_0^t dt\, \mathbf{V} \cdot [\mathbf{V}(\partial \rho / \partial t + \mathbf{V}(\mathbf{V} \cdot \mathrm{grad}\rho)) + \rho(\partial \mathbf{V}/\partial t + \mathbf{V} \cdot \mathrm{grad}\mathbf{V})]$

$= \int_0^t dt \{D(\tfrac{1}{2}\rho \mathbf{V}^2)/Dt + \tfrac{1}{2}\mathbf{V}^2 (D\rho/Dt) - \tfrac{1}{2}\rho (D\mathbf{V}^2/Dt) + \rho \mathbf{V} \cdot (D\mathbf{V}/Dt\}$.

(b) $L \equiv \int_0^t dt\, \mathbf{V} \cdot \mu \nabla^2 \mathbf{V} = \mu \int_0^t dt [\mathrm{div}(\mathbf{V}\mathrm{div}\mathbf{V} - \mathbf{V} \times \mathrm{curl}\mathbf{V}) - (\mathrm{div}\mathbf{V})^2 - (\mathrm{curl}\mathbf{V})^2]$

$= -\mu \int_0^t dt [(\mathrm{div}\mathbf{V})^2 + (\mathrm{curl}\mathbf{V})^2]$

L ≡ (vanishing by surface integral)

−(**thermal loss by jet blow**)−(**thermal loss by fluid stiring**).

＊$\nabla^2 \mathbf{V} = \mathrm{graddiv}\mathbf{V} - \mathrm{curlcurl}\mathbf{V}$. ＊$\mathrm{div}(\mathbf{A} \times \mathbf{B}) = \mathbf{B}\,\mathrm{curl}\mathbf{A} - \mathbf{A}\,\mathrm{curl}\mathbf{B}$.

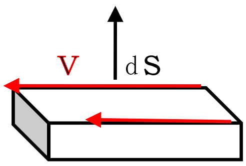
$-\iiint dV.\, \mathbf{V}\mathrm{curlcurl}\mathbf{V} = -\iiint dV \{\mathrm{div}(\mathrm{curl}\mathbf{V} \times \mathbf{B}) + \mathrm{curl}\mathbf{V}.\, \mathrm{curl}\mathbf{V}\}$.

$\iiint dV.\, \mathbf{V}\mathrm{graddiv}\mathbf{V} = \iiint dV \{\mathrm{div}(\mathbf{V}\mathrm{div}\mathbf{V}) - \mathrm{div}\mathbf{V}.\, \mathrm{div}\mathbf{V}\}$.

Thus energy from **friction term** become negative,which mean **energy absorbing** from left momentum energy and the actuality is **dissipative heat** by **frictional turbulence**.

Note this is just the origin of **chaos** in NS equation !!.

$*\oiiint dV\,\mathrm{div}(\mathbf{V}\,\mathrm{div}\mathbf{V}-\mathbf{V}\times\mathrm{curl}\mathbf{V})=\oint d\mathbf{S}\,(\mathbf{V}\,\mathrm{div}\mathbf{V}-\mathbf{V}\times\mathrm{curl}\mathbf{V})=0.$



Boundary surface flow $\mathbf{V}$ is orthogonal to $d\mathbf{S}$,so 1st term is zero. curl$\mathbf{V}$ is parallel with $d\mathbf{S}$, and $\mathbf{V}\times\mathrm{curl}\mathbf{V}$ is orthogonal to $d\mathbf{S}$, so 2nd term is also zero.

(c)$M\equiv-\int_0^t dt\,\mathbf{V}\cdot\mathrm{grad}P=-\int_0^t dt[\mathrm{div}(\mathbf{V}\cdot P)-P\,\mathrm{div}\mathbf{V}].$

$M\equiv$(1st term is net work by pressure on a closed surface integral and 2nd one is volume integral energy jet flow($\mathrm{div}\mathbf{V}$)with pressure).

*What causes pressure ? in ocean and atmosphere flows,

(d)$N\equiv\int_0^t dt\,\mathbf{V}\rho\mathbf{g}$. <This term could be potential energy of ρgh,where h is height(depth)>. This is a work by **gravity** with parallel component of $\mathbf{V}$. This is a vertical flow due to **sink**(ρ change by dense **salinity** in sea ice freezing),or ρ change by **cold heat**(**convection)**) or **buoyancy** force(density ρ change by **hot heat**(**convection)**).

[3] : Entropy Generating in NS-Fluid Equation of unknown variables{$\mathbf{V}$, P, ρ, T, Q}.<independent eqns{**NS**,(2)(3)(4)(5)}>

-The simultaneous equations-

$E(t)-E(0)=\int_0^t dW=\int_0^t dt\,\mathbf{V}\cdot\mathbf{f}.$

Fluid dynamical energy relation$\equiv W$:

$dW/dt=\mathbf{V}\cdot D(\rho\mathbf{V})/Dt=\mathbf{V}\cdot\mu\nabla^2\mathbf{V}-\mathbf{V}\cdot\mathrm{grad}P+\mathbf{V}\cdot\rho\mathbf{g}.$

$=-\mu[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]-\mathbf{V}\cdot\mathrm{grad}P+\mathbf{V}\cdot\rho\mathbf{g}.$

Thermal energy relation$\equiv Q$:

$\partial Q(\mathbf{x},t)/\partial t=-\mathrm{div}(Q\mathbf{V})+J_i+\lambda(\mathbf{x},t)\,\mathrm{grad}T(\mathbf{x},t)+J_R(\mathbf{x},t).$

$J_i\equiv+\mu[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2].$

Total Energy Relation:Energy Conservation Low in NS Fluid-Equation $E \equiv W + Q$.

$(d/dt)[W+Q] = -\mathbf{V} \cdot \mathrm{grad}\,P + \mathbf{V} \cdot \rho\,\mathbf{g} - \mathrm{div}(Q\,V) + \lambda(\mathbf{x}, t)\,\mathrm{grad}\,T(\mathbf{x}, t) + J_R(\mathbf{x}, t)$.

system energy = {pressure + gravity potential} + {convection + conduction + radiation}heats

(1)**2nd low of Thermodynamics<entropy density increasing>:**

$dS = dQ/T$. $dS/dt = (dQ/dt)/T$.

(2)**Thermodynamical state equation of fluid(ideal gas):**

$PV = nRT$.   R≡gas constant. M=molecular mass.

$P = (m/VM)RT = (\rho/M)RT$.

(3)**Specific heat** C_P **of fluid matter.**

$Q(\mathbf{x}, t) = C_P T(\mathbf{x}, t)$.

(4)**Heat flow conservation low:**

$\partial Q(\mathbf{x}, t)/\partial t = J_i$ {**internal heat generation** by $\mu \nabla^2 V$} +

+ J{heat flows by external **radiation,conduction and convection**}

$= J_i - \mathrm{div}(Q\,V) + J$ {heat input by external **radiation,conduction**}.

(a)**radiation** $\equiv J_R(\mathbf{x}, t)$: author don't know the explicit form !

(b)**conduction** $\equiv J_{cd}(\mathbf{x}, t)$: $\lambda(\mathbf{x}, t)\,\mathrm{grad}\,T(\mathbf{x}, t)$

(c)**convection** $\equiv J_{cv}(\mathbf{x}, t)$: $-\mathrm{div}(Q(\mathbf{x}, t)\,\mathbf{V}(\mathbf{x}, t))$

(5)**fluid mass(ocean or atmosphere)density** ρ **conservation low:**

$\partial \rho(\mathbf{x}, t)/\partial t == -\mathrm{div}(\rho\,\mathbf{V}) + J_m\{?\}$.

Those simultaneous equations{**NS**,(2)(3)(4)(5)}could determine unknown variables{V, P, ρ, T, Q}.Thus we could calculate (dS/dt) **the entropy increasing**.

[4] : Information measure in NS-Fluid Equation.

Reference:*Le'on Brillouin,Science and Information Theory,Academic Press,N.Y,1956,1962.*

In this book,Brillouin described "**man observing information**" is measured by negative entropy.Therefore entropy increasing in NS Fluid Equation is not reversible,but **irreversible generating positive entropy** which means nothing but "**information loss**" in Fluid equation solution(**Chaos**).Therefore long time prediction could not be accomplished without reducing friction term $\mu \nabla^2 V$, which could be done by so called **scale transformation in time and space variables**.

APPENDIX_1:

How to transform Fluid Equation more causalitical !! 2013-3-27,9-26.

Large space and time scale view by scaling transformation(Reinolds analogy low).

So called Chaos in fluid equation is due to friction term $\mu \nabla^2 \mathbf{u}$, which causes long term weather prediction difficult.However a method could overcome the difficulty.
Fluid equation could be transformed into new space and time variables{x',t'} from {x,t} by $\{x \equiv Lx',\ t \equiv (L/U)t',\ \rho \equiv \rho';\ m \equiv L^3 m',\ \mathbf{\Omega} \equiv \mathbf{\Omega}'\}$

$u \equiv \partial x/\partial t = \partial(Lx')/\partial((L/U)t') = u\,\partial x'/\partial t' = Uu'.$

$\alpha \equiv \partial u/\partial t = \partial(Uu')/\partial((L/U)t') = (U^2/L)\,\partial u'/\partial t' = (U^2/L)\,\alpha'.$

$P \equiv f/x^2 = m\alpha/x^2 = L^3 m'(U^2/L)\,\alpha'/(Lx')^2 = U^2 m'\,\alpha'/x'^2 = U^2 p'.$

$\partial P/\partial x = (U^2/L)\,\partial p'/\partial x'.$

$\rho \equiv m/x^3 = L^3 m'/(Lx')^3 = m'/x'^3 = \rho'.$

$\partial^2 u/\partial x^2 = \partial^2(Uu')/\partial(Lx')^2 = (U/L^2)\,\partial^2 u'/\partial x'^2$

$-2\rho\,\mathbf{\Omega}\times\mathbf{u} = -(U)2\rho'\,\mathbf{\Omega}'\times\mathbf{u}'$

$\rho\{\partial\mathbf{u}/\partial t + \mathbf{u}.\mathrm{grad}\mathbf{u}\} = -\mathrm{grad}P + \mu\nabla^2\mathbf{u} - 2\rho\,\mathbf{\Omega}\times\mathbf{u} + \rho\mathbf{g}.$

$\rho'\{(U^2/L)\,\partial\mathbf{u}'/\partial t' + (U^2/L)\ \mathbf{u}'.\mathrm{grad}\mathbf{u}'\}$

$= -(U^2/L)\,\mathrm{grad}'P' + \mu(U/L^2)\nabla'^2\mathbf{u}' - (U)2\rho\,\mathbf{\Omega}\times\mathbf{u} + \mathbf{g}\rho'.$

$\rho'\{\partial\mathbf{u}'/\partial t' + \mathbf{u}'.\mathrm{grad}'\mathbf{u}'\}$

$= -\mathrm{grad}'P' + (\mu/UL)\nabla'^2\mathbf{u}' - 2\rho'(L/U)\,\mathbf{\Omega}'\times\mathbf{u}' + \rho'(L/U^2)\mathbf{g}$

$= -\mathrm{grad}'(P/U^2) + (\mu/UL)\nabla'^2\mathbf{u}' - 2\rho'\,\underline{(L/U)\,\mathbf{\Omega}}\times\mathbf{u}' + \rho'\,\underline{(L/U^2)\mathbf{g}}.$

Then taking U=1,L=larger could make(μ /UL)≡ <u>μ ' smaller(almost nothing friction force)</u>.
<u>grad'**u**', **Pressure gradient**, **Coliori force** and **gravity** become larger</u>.
This is to make fulid equation more causalitical one in **large space and time scale view**.
This might be a certification for global climate model calculation.
example) :
60km(man size)→$6\mathrm{x}10^6$m(earth size)⇒U≡1;L≡10^2.

$\mu = 1.307\mathrm{x}10^{-3}\,\mathrm{Ns/m^2}) \rightarrow \mu' = \mu/UL = 1.307\mathrm{x}10^{-5}\,\mathrm{Ns/m^2}.$

$\rho'\{\partial\mathbf{u}'/\partial t' + \mathbf{u}'.\mathrm{grad}\mathbf{u}'\} = -\mathrm{grad}'(P') - 2\rho'\,\underline{(L/U)\,\mathbf{\Omega}}\times\mathbf{u}' + \rho'\,\underline{(L/U^2)\mathbf{g}}.$

In order to be **complete dynamic equation** ,left term need +u$\{\partial\rho/\partial t + \mathbf{u}.\mathrm{grad}\,\rho\}$for force by density ρ' change.

Appendix _2 : :vector calculus in left term.

$D(\rho\mathbf{V})/Dt=\partial_t(\rho\mathbf{V})+\langle\mathbf{V}\cdot\mathrm{grad}(\rho\mathbf{V})\rangle$

$=\partial_t(\rho\mathbf{V})+\langle\mathbf{V}\cdot(\mathrm{grad}(\rho\mathbf{V}))\rangle=\partial_t(\rho\mathbf{V})+\mathbf{V}\langle\mathbf{V}.\mathrm{grad}\,\rho\rangle+\rho\langle\mathbf{V}\cdot\mathrm{grad}\mathbf{V}\rangle$

$=\underline{\rho[\partial_t\mathbf{V}+\langle\mathbf{V}\cdot\mathrm{grad}\mathbf{V}\rangle]}+\mathbf{V}[\partial_t\rho+\langle\mathbf{V}\cdot\mathrm{grad}\,\rho\rangle]$.......OK

$*(\mathrm{grad}(\rho\mathbf{V})\equiv[\mathrm{grad}(\rho V_1),\mathrm{grad}(\rho V_2),\mathrm{grad}(\rho V_3)]$

$=[V_1\mathrm{grad}\,\rho+\rho\,\mathrm{grad}V_1,\ V_2\mathrm{grad}\,\rho+\rho\,\mathrm{grad}V_2,\ V_3\mathrm{grad}\,\rho+\rho\,\mathrm{grad}V_3]$

$=[V_1\mathrm{grad}\,\rho+\rho\,\mathrm{grad}V_1,\ V_2\mathrm{grad}\,\rho+\rho\,\mathrm{grad}V_2,\ V_3\mathrm{grad}\,\rho+\rho\,\mathrm{grad}V_3]$

$\mathbf{V}\cdot(\mathrm{grad}(\rho\mathbf{V}))=\mathbf{V}\langle\mathbf{V}.\mathrm{grad}\,\rho\rangle+\rho\langle\mathbf{V}\cdot\mathrm{grad}\mathbf{V}\rangle.$ .....OK

$**\ \underline{\rho\mathbf{V}\cdot(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})}=\langle\mathbf{V}\cdot\langle\mathbf{V}\cdot(\mathrm{grad}(\rho\mathbf{V}))\rangle\rangle-\mathbf{V}^2\langle\mathbf{V}.\mathrm{grad}\,\rho\rangle$

$K\equiv\mathbf{V}\cdot[\mathbf{V}(\partial\rho/\partial t+(\mathbf{V}\cdot\mathrm{grad}\,\rho))+\rho(\partial\mathbf{V}/\partial t+\mathbf{V}\cdot\mathrm{grad}\mathbf{V})]$

$=\mathbf{V}^2(\partial\rho/\partial t)+(\mathbf{V}\cdot\mathrm{grad}\,\rho))+\tfrac{1}{2}\rho\,\partial_t(\mathbf{V}^2)+\rho\mathbf{V}\cdot(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})$

$=\partial_t(\tfrac{1}{2}\rho\mathbf{V}^2)+\tfrac{1}{2}\mathbf{V}^2\partial_t\rho+\mathbf{V}^2(\mathbf{V}\cdot\mathrm{grad}\,\rho))+\rho\mathbf{V}\cdot(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})$

$=\underline{\partial_t(\tfrac{1}{2}\rho\mathbf{V}^2)+(\tfrac{1}{2}\mathbf{V}^2)\langle\mathbf{V}\cdot\mathrm{grad}\,\rho\rangle+\rho\mathbf{V}^2\mathrm{div}\mathbf{V}}$

$+\tfrac{1}{2}\mathbf{V}^2\partial_t\rho+\mathbf{V}^2(\mathbf{V}\cdot\mathrm{grad}\,\rho))+\rho\mathbf{V}\cdot(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})$

$-(\tfrac{1}{2}\mathbf{V}^2)\langle\mathbf{V}\cdot\mathrm{grad}\,\rho\rangle-\rho\mathbf{V}^2\mathrm{div}\mathbf{V}$

$=D((\tfrac{1}{2}\rho\mathbf{V}^2)/Dt+\underline{\tfrac{1}{2}\mathbf{V}^2\partial_t\rho+(\tfrac{1}{2}\mathbf{V}^2)\langle\mathbf{V}\cdot\mathrm{grad}\,\rho\rangle}+\rho\mathbf{V}\cdot(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})-\rho\mathbf{V}^2\mathrm{div}\mathbf{V}$

$=D((\tfrac{1}{2}\rho\mathbf{V}^2)/Dt+\tfrac{1}{2}\mathbf{V}^2D\rho/Dt+\rho\mathbf{V}\cdot[(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})-\mathbf{V}\mathrm{div}\mathbf{V}]$

$D((\tfrac{1}{2}\rho\mathbf{V}^2)/Dt=\partial_t(\tfrac{1}{2}\rho\mathbf{V}^2)+\langle\mathbf{V}\cdot\mathrm{grad}(\tfrac{1}{2}\rho\mathbf{V}^2)\rangle$

$=\partial_t(\tfrac{1}{2}\rho\mathbf{V}^2)+\langle\mathbf{V}\cdot[(\tfrac{1}{2}\mathbf{V}^2\cdot\mathrm{grad}\,\rho)+\rho\mathbf{V}\mathrm{div}\mathbf{V}]\rangle$

$=\partial_t(\tfrac{1}{2}\rho\mathbf{V}^2)+(\tfrac{1}{2}\mathbf{V}^2)\langle\mathbf{V}\cdot\mathrm{grad}\,\rho\rangle+\rho\mathbf{V}^2\mathrm{div}\mathbf{V}.$ ......OK

$\mathrm{grad}\mathbf{V}^2=2\mathbf{V}\mathrm{div}\mathbf{V}.$

$D\mathbf{V}^2/Dt=\partial_t\mathbf{V}^2+\langle\mathbf{V}\cdot\mathrm{grad}\mathbf{V}^2\rangle=\partial_t\mathbf{V}^2+\langle\mathbf{V}\cdot2\mathbf{V}\mathrm{div}\mathbf{V}\rangle=\partial_t\mathbf{V}^2+2\mathbf{V}^2\mathrm{div}\mathbf{V}.$

$=2\mathbf{V}\partial_t\mathbf{V}+2\mathbf{V}^2\mathrm{div}\mathbf{V}=2\mathbf{V}(\partial_t\mathbf{V}+\mathbf{V}\mathrm{div}\mathbf{V}).$

$\tfrac{1}{2}D\mathbf{V}^2/Dt=\mathbf{V}(\partial_t\mathbf{V}+\mathbf{V}\mathrm{div}\mathbf{V}).$

$\tfrac{1}{2}\rho D\mathbf{V}^2/Dt=\rho\mathbf{V}(\partial_t\mathbf{V}+\mathbf{V}\mathrm{div}\mathbf{V}).$

$-\rho\mathbf{V}(\mathbf{V}\mathrm{div}\mathbf{V})=\rho\mathbf{V}\partial_t\mathbf{V}-\frac{1}{2}\rho D\mathbf{V}^2/Dt$

$(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})=D\mathbf{V}/Dt-\partial_t\mathbf{V}$

$\rho\mathbf{V}\cdot(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})=\rho\mathbf{V}\cdot(D\mathbf{V}/Dt-\partial_t\mathbf{V})$

$\rho\mathbf{V}\cdot[(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})-\mathbf{V}\mathrm{div}\mathbf{V}]=-\frac{1}{2}\rho D\mathbf{V}^2/Dt+\rho\mathbf{V}\cdot(D\mathbf{V}/Dt$

$K=D((\frac{1}{2}\rho\mathbf{V}^2)/Dt+\frac{1}{2}\mathbf{V}^2D\rho/Dt+\rho\mathbf{V}\cdot[(\mathbf{V}\cdot\mathrm{grad}\mathbf{V})-\mathbf{V}\mathrm{div}\mathbf{V}]$

$=D(\frac{1}{2}\rho\mathbf{V}^2)/Dt+\frac{1}{2}\mathbf{V}^2(D\rho/Dt)-\frac{1}{2}\rho(D\mathbf{V}^2/Dt)+\rho\mathbf{V}\cdot(D\mathbf{V}/Dt).$

＊Now author don’t know well about the physical meanings.