

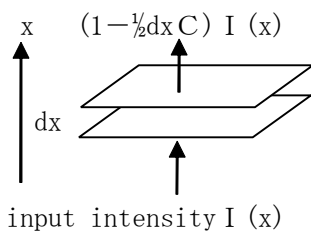
'08/12/2, 09/10/8,

RADIATIVE FORCING (the general formulation and interpretation)-rev1.

Radiative Forcing(RF) is surplus energy flow(W/m²) into global land-sea surface:
 $RF = \sigma [T_A(t)^4 - T_G(t)^4]$. $T_G(t)$ is global temperature, while $T_A(t)$ is so called precedent one as $(F_0/4)(1-m)a = \sigma T_A(t)^4$: fuction of albedo= $m(t)$ at now and $a(t)$ = passing probability of cooling radiation into cosmic space from terrestrial surface of $T_G(t)$. GHG concentration at now rules $a(t)$.

[1]: Passing porobability of cooling radiation due to collision cross-section :

Concerned molecule gas global number "N" is defined in atmospehre volume V_A ($C = N v_0 / V_A \equiv$ gas concentration with molecule volume= v_0). Assume differential segment layers of atmosphere at height $(x, x+dx)$. Then $S dx$ is differential volume and $C S dx$ is molecule volume with the area = $C S$. Hence **mean collision cross section** of molecules at $(x, x+dx)$ is C /unit area.



- (1) direct passing probability = $1 - dx C$
- (2) trapped and upward-radiation = $\frac{1}{2} dx C$
- (3) total passing probability = $1 - \frac{1}{2} dx C$
- (4) $dI = -\frac{1}{2} dx C (x) I(x)$.

☞: Our concern is upward and downward probability only with assuming nothing non-equilibrium accumulation of radiation in atmosphere.

(5) $dI/dx = -\frac{1}{2} C(x) I(x)$. $\rightarrow P(x) = I(x)/I(0) = \exp\langle -\frac{1}{2} \int_0^x dy C(y) \rangle$.
 $\rightarrow P(\infty) = I(\infty)/I(0) = \exp\langle -\frac{1}{2} \int_0^\infty dy C(y) \rangle \equiv \exp[-\frac{1}{2} \langle h \rangle C]$.

Upper limit of integral domain in (5) need effective atmospheric height $\equiv \langle h \rangle$.

Half the atmosphere is within 6 km of the surface, 90% is within 20 km.

<http://www.exo.net/~pauld/workshops/Greenhouse%20Effect/greenhouse.html>

(6) $C = V_C/V_A = N v_0/V_A =$ particle number \times molecule volume/volume V_A . 1ppm = $10^{-6}/uv$.

(a) Ideal gas has same number of molecules in same volume : 6.02x10²⁶/22.4litter.

* molecule volume $\equiv v_0 = 22.4 \times 10^{-3} m^3 / 6.02 \times 10^{26} = 3.72 \times 10^{-29} m^3 = (3.34 \times 10^{-10} m)^3$.

* Number of particles in 1ppm/m³ $\equiv n_0 = 10^{-6} / 3.72 \times 10^{-29} m^3 = 2.69 \times 10^{22} / m^3$.

CO2 (1.9ppm) weight//m³ = $(44g/6.02 \times 10^{23}) \times 1.9 \times 2.69 \times 10^{22} / m^3$.

(b) total air weight $\equiv M_T = P(g \times 1atm) \times 4 \pi R_E^2 / g = 1033 kg \times 4 \pi (6.38 \times 10^6 m)^2 = 5.28 \times 10^{17} kg$

[2]: Taking molecule spectroscopic character for collision cross-section :

(1) T^oC Blackbody radiation from the global surface (x=0) as input for GHG :

(a) point spectrum density function: $\rho(x=0; T; \nu = E/h) = (8 \pi \nu^2 / c^3) / [\exp(h \nu / kT) - 1]$.

(b) energy flux $\equiv I (W/m^2) \equiv \int_0^\infty dE. c \rho / 4 = (c/4) \int_0^\infty dE. E (8 \pi E^2 / h^3 c^3) / [\exp(h \nu / kT) - 1]$
 $= (8 \pi c / 4 \beta^4 h^3 c^3) \int_0^\infty d(\beta E). (\beta E)^3 / [\exp(\beta E) - 1] = ([2 \pi^5 / 15] k_B^4 / h^3 c^3) T^4 = \sigma T^4.$
 $= [\pi^4 / 15], \beta \equiv 1/k_B T.$

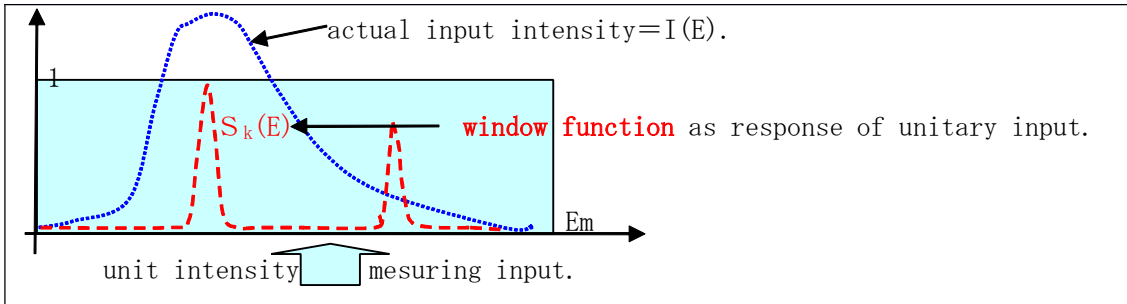
(c) mean outflow of energy spectrum from unit area of land & sea :

$$I(T; E=h\nu) \equiv (c/4) \rho(T; E=h\nu) = (2 \pi E^3 / h^3 c^2) / [\exp(E/kT) - 1].$$

(2) Absorb(trapping) window function = $S_k(E)$ of molecule "k" .

(a) absorption rate with unit input intensity = $S_k(E)$:

$$0 \leq S_k(E) \leq 1 \text{ (window function (trapping rate))}.$$



(b) $p_k = \text{trapp probability} = \int_0^{E_m} dE S_k(E) I(E) / \int_0^{E_m} dE. I(E).$

$$d p_k(E) = dE S_k(E) I(E) / \int_0^{E_m} dE. I(E).$$

(3) collision cross section, volume concentration = C_k , paticle number = N_k :

$$C_k \equiv N_k v_0 / V_{A_0} \quad V_A = 4 \pi R_E^2 \times h_A = 4 \pi (\text{earth radius})^2 \times (\text{height of atmosphere}).$$

$$C_k \equiv N_k v_0 / 4 \pi R_E^2 h_A. \text{ <atmospheric concentration>}$$

$$dx S C_k = \text{differential vol.} \rightarrow dx C_k = (\text{cross section/unit area}) \text{ for } dx.$$

Discussion in [1] is case of $S_k(E) \equiv 1$. Therefore,

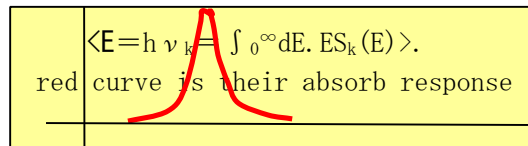
(4) spectroscopic collision cross section depending energy E.

$$dI(E; x) \cdot dE / I(E; x) = -\frac{1}{2} [dx C(E; x)] dE = -\frac{1}{2} [dx C_k(x) \cdot S_k(E)] dE. \dots \text{ <probability>}$$

$$\rightarrow dI(E; x) = -\frac{1}{2} [dx C_k(x) S_k(E)] I(E; x). \text{ <trapp probility with the window-function>}$$

This discussion will be mentioned again in [3]. Below (5)(6)(7)(8) are don't care.

(5) At a time, a molecule "k" absorb photons at **resonance** $\langle E = h \nu \rangle$ with the window function $S_k(E)$.



$E = h \nu_k$ should be as $\int_0^\infty dE. E. S_k(E)$ for single molecule.

(6) mean life time τ_k of excited level at $E = h \nu_k$. <time & energy uncertainty eqn>

half width of spectrum $= h \nu_k / Q_k$. ($1 \ll Q_k \doteq 10$)

$$\tau_k \times (h \nu_k / Q_k) \doteq h / 2 \pi. \rightarrow \tau_k \doteq Q_k / 2 \pi \nu_k.$$

$$\rightarrow P_{\max} = h \nu_k / \tau_k \doteq 2 \pi h \nu_k^2 / Q_k \doteq h \nu_k^2. \langle \text{maximum trapping power rate (W=J/s)} \rangle$$

Example) CO2 first resonance peak at 1300nm = 2.31×10^{14} Hz.

$$h \nu_k^2 = 6.63 \times 10^{-34} \text{ J s} \times (2.1 \times 10^{14})^2 = 2.9 \times 10^{-5} \text{ W.}$$

$$N_A \times h \nu_k^2 = 6.02 \times 10^{26} \times 2.9 \times 10^{-5} \text{ W. It's outrageous power by mere 1 mol CO}_2.$$

In this way, you could verify **nothing saturation** in GHG heat trapping.

GHG is "linear" band trapping filter causing **downward reradiation** of cooling radiation from terrestrial surface.

Carbon dioxide spectrum:

<http://www.iitap.iastate.edu/gccourse/forcing/spectrum.html>

<http://webbook.nist.gov/cgi/cbook.cgi?ID=C124389&Units=SI&Type=IR-SPEC&Index=1#IR-SPEC>

(7) **Band Pass Filter as collision cross sectional :**

$$I(E) \rightleftharpoons \boxed{\Gamma(E)} \rightleftharpoons J(E) = \Gamma(E) I(E). \quad P_0 = \int_0^\infty dE. J(E).$$

Note that band pass (BPF) is view from **collision cross section**.

Now we discuss on **passing probability** of terrestrial radiation toward space. Note that, in [1]:(5)', concentration "C" is an **absorption intensity** as collision cross section, so Band Pass Filtering $S_k(E)$ should be multiplied with " $C_k \cdot S_k(E)$ ".

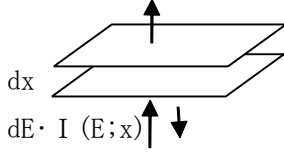
(8) **Passing probability spectrum of cooling radiation from the surface:**

See <http://en.wikipedia.org/wiki/Image:Atmosfaerisk_spredning.gif>.

The graph is output response as **Band Pass Filter (white region)** of the GHG with normalized input intensity. Blue part is trapped by GHG. Then methane act at $7 \sim 8 \mu$, where is almost **white region**, so it is to act effectively by few concentration.

**[3]: Passing probability as collision cross sections
with spectroscopic characteristic of GHGs.**

$$(1) [1 - \frac{1}{2} dx C(E; x)] dE \cdot I(E; x)$$



- (1) through probability = $1 - dx C$.
- (2) trapped and upward radiation = $\frac{1}{2} dx C$.
- (3) total penetration probability = $1 - \frac{1}{2} dx C$.
- (4) $dI(E; x) = -\frac{1}{2} dx C(E; x) I(E; x)$.

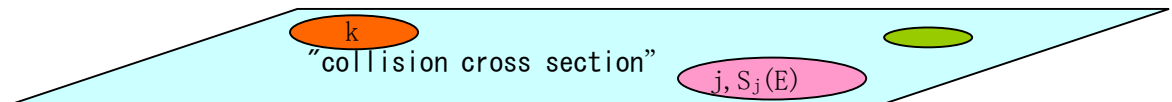
$$(2) dI(E; x) = -\frac{1}{2} dx C(E; x) I(E; x) \rightarrow dI(E; x)/dx = -\frac{1}{2} C(E; x) I(E; x)$$

$$\rightarrow I(E; x) = I(E, 0) \exp\left\langle -\frac{1}{2} \int_0^x dy C(E; y) \right\rangle$$

$$J = \int_0^\infty dE \cdot I(E; x) = \int_0^\infty dE \cdot I(E, 0) \exp\left\langle -\frac{1}{2} \int_0^x dy C(E; y) \right\rangle$$

$$a = \text{total passing probability} = \int_0^\infty dE \cdot I(E, 0) \exp\left\langle -\frac{1}{2} \int_0^x dy C(E; y) \right\rangle / \int_0^\infty dE \cdot I(E, 0)$$

$$(3) C(E; x) = \alpha / \langle h \rangle \sum_{k=1}^N C_k(x) S_k(E) = \text{Sum}\{ \text{"k" concentration} \times \text{BPF spectroscopic density} \}$$



$C_k(x) \equiv$ collision cross section proportional to volume concentration of "k" at x.

$S_k(E)$: Spectroscopic window function or band pass filter transfer function.

$\alpha / \langle h \rangle \equiv$ Normalization constant with mean atmospheric height $\langle h \rangle$.

$C_k =$ GHG concentration of molecule "k".

(4) a \equiv passing probability in total GHGs :

$a(x) =$ total passing probability

$$= \int_0^\infty dE \cdot I(E, 0) \exp\left\langle -\frac{1}{2} \alpha / \langle h \rangle \int_0^x dy \sum_{k=1}^N C_k(y) S_k(E) \right\rangle / \int_0^\infty dE \cdot I(E, 0)$$

$$= \int_0^\infty dE \cdot I(E, 0) \prod_{k=1}^N \exp\left\langle -\frac{1}{2} \alpha S_k(E) \int_0^x dy C_k(y) \right\rangle / \langle h \rangle \int_0^\infty dE \cdot I(E, 0)$$

$$a \equiv a(x=\infty) \equiv \left[\int_0^\infty dE \cdot I(E, 0) \right]^{-1} \int_0^\infty dE \cdot I(E, 0) \prod_{k=1}^N \exp\left[-\frac{1}{2} \alpha S_k(E) C_k \right]$$

$$\equiv N_a \int_0^\infty dE \cdot I(E, 0) \prod_{k=1}^N G_k(E)$$

$b = (1-a)$ is trapp and re-radiation probability. "a" is determined by "molecule spectroscopic character $S_k(E)$ and the concentration $C_k(x)$ in atmosphere at x".

GHG acts as cutting shape off of cooling radiation spector $I(E, 0)$ by absorbtion characteristic of $S_k(E)$. Total effects of each molecules is equivalent to **cascade filters**.

As shall be mentioned in later, low concentration acts high sensitivity of RF (radiation force) in **nothing or less trapping energy band** of such as CH4.

$$I(E, 0) \rightarrow \boxed{G_1(E) \rightarrow G_2(E) \rightarrow G_3(E) \rightarrow \dots \rightarrow \dots \rightarrow G_N(E)} \rightarrow J(E, 0) = \Gamma(E) I(E, 0)$$

$$\Gamma(E) = \prod_{k=1}^N \exp\left[-\frac{1}{2} \alpha S_k(E) C_k \right]$$

(5) $G_k(E) \equiv \exp[-\frac{1}{2} \alpha S_k(E) C_k]$: Attenuation weight function.

$\rightarrow \partial G_k(E) / \partial C_k = -\frac{1}{2} \alpha S_k(E) G_k(E; C_k)$. [high sensitivity at low concentration]

$$\partial a / \partial C_k = \left[\int_0^\infty dE \cdot I(E, 0) \right]^{-1} \int_0^\infty dE \cdot I(E, 0) \Pi_{j \neq k=1}^N G_j(E) \times \left[\partial G_k / \partial C_k \right]$$

$$= N_a \int_0^\infty dE \cdot \langle \langle -\frac{1}{2} \alpha S_k(E) \rangle \rangle I(E, 0) \Pi_{j=1}^N G_j(E).$$

$$\doteq \langle \langle -\frac{1}{2} \alpha S_k(E) \rangle \rangle N_a \int_0^\infty dE \cdot I(E, 0) \Pi_{j=1}^N G_j(E).$$

* the last eqn is due to theorem: $\int_0^\infty dE \cdot A(E) B(E) = A(E) \int_0^\infty dE \cdot B(E)$.

$$* \partial a(t) / \partial C_k(t) = -\frac{1}{2} \alpha S_k(E) a(t).$$

If $-\frac{1}{2} \alpha S_k(E)$ were a constant, then it could be easier.

[4]: GHG concentrations and the (global) Radiative Forcing:

(1) Global energy balance equation:

$$(a) \pi R_E^2 F_0 [1-m] / [1-b] = 4 \pi R_E^2 \sigma T^4. \quad \langle R_E \equiv \text{earth radius} = 6.38 \times 10^6 \text{m} \rangle.$$

<heat input = heat output: heat budget in equilibrium global temperature>

$$(b) F \equiv (F_0/4) [1-m(t)] / a(t) = \sigma T_A(t)^4.$$

$$\rightarrow \partial F / \partial a = -(F_0/4) (1-m) / a^2 = -\sigma T_A^4 / a.$$

Actually, the equation has a **deficit** causing $T_G(t)$ rise induced by larger left term $\equiv (F_0/4) [1-m(t)] / [1-b(t)] \equiv \sigma T_A(t)^4$, which is a function of current albedo $=m(t)$ and (cooling radiation) passing probability $a(t) = [1-b(t)]$, while right term is ruled by the outrageous global heat capacity that make $T_G(t)$ rise slowly.

(2) RADIATIVE FORCING ($\delta F \equiv RF$):

$$\delta F(t) \equiv \sigma [T_A(t)^4 - T_G(t)^4] \equiv (F_0/4) [1-m(t)] / [1-b(t)] - \sigma T_G(t)^4.$$

$$C_G (dT_G(t) / dt) = 4 \pi R_E^2 \delta F(t). \quad \langle \langle C_G \equiv \text{global heat capacity} = 2.7 \times 10^{23} \text{J/K} \rangle \rangle$$

See this <http://www.geocities.jp/sqkh5981g/EGT.pdf>. $\delta F \equiv F_{in} - F_{out}$ is the deficit definition of radiative forcing at top of troposphere, which could not help to heat up globe by $C_G (dT_G(t) / dt)$. $\delta F = 1.6 \text{W/m}^2$ just agrees with current temperature rise $(dT_G(t) / dt) = 0.06 \text{K/y}$. $\langle \langle \text{the definition by IPCC (1750) is not correct!!} \rangle \rangle$

(3)Sensitivity of RF by GHG concentration : $\partial \delta F / \partial C_k = (\partial F / \partial a) (\partial a / \partial C_k)$.

$$= -F_0(1-m)/a^2 \times -\frac{1}{2} \alpha S_k("E") > a = F_0(1-m)/a \times \frac{1}{2} \alpha S_k("E") = \frac{1}{2} \alpha S_k("E") \sigma T_A(t)^4.$$

$$\Rightarrow : \langle [\partial G_k / \partial C_k] / G_k \rangle = [\partial / \partial C_k] \ln G_k = -\frac{1}{2} \alpha S_k(E) \circ$$

-Sensitivity of RF by GHG concentration : - :

$$\partial \delta F / \partial C_k = \frac{1}{2} \alpha S_k("E") \sigma T_A(t)^4.$$

\Rightarrow : $a = N_a \int_0^\infty dE \cdot I(E, 0) \prod_{k=1}^N G_k(E)$ is passing probability of input $I(E, 0)$.

Masking(filtering) function = $\prod_{k=1}^N G_k(E)$.

[5]:Summary:

$$\delta F(t) \equiv (F_0/4) [1-m(t)]/a(t) - \sigma T_G(t)^4. \rightarrow C_G(dT_G(t)/dt) = 4 \pi R_E^2 \delta F(t).$$



$m(t)$: current global albedo.

$a(t) = a(\{C_k(t) | k=1, 2, \dots\})$: pass probability of cooling radiation, ruled by current GHG concentrations.

Postscript at '09/10/8: This is a cut and try instant version, so author wish more correction if possible.