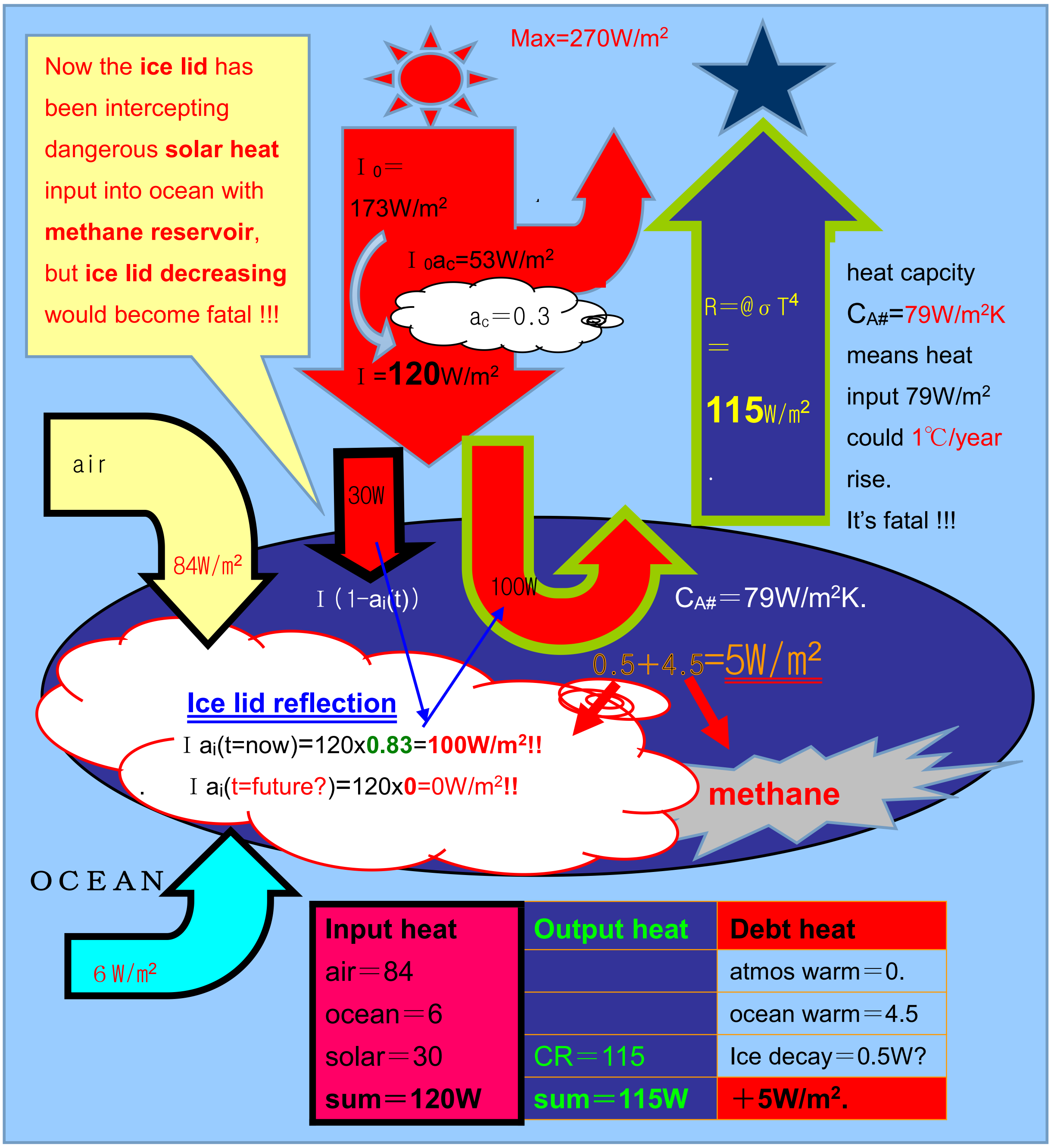
Rapid Temeperature Rise in Arctic_a simple verification. 2014/4/15,18,25,5/13

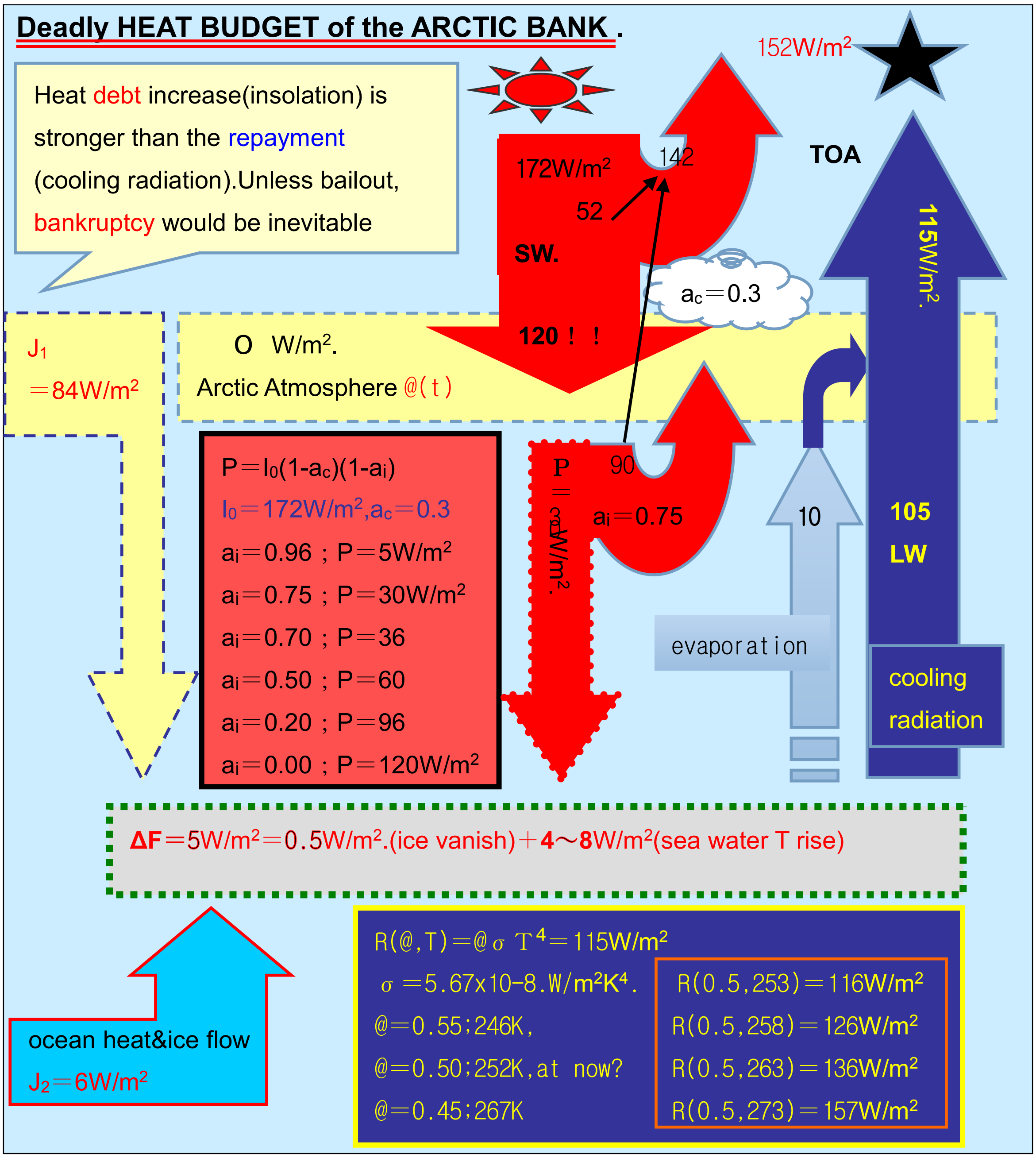
Arctic has been warming due to beginning of ice lid vanishing.The cooling is only by natural radiation from surface to cosmos.Temperature rise could increase the radiation,though **ice lid vanishing rate is far stronger than** **the radiation increasing**.Coming abrupt Arctic sea water temperature rise would be fatal to trigger Arctic methane catastrophe.

[０]：**What,Why,When,and How as for Arctic<Annual Heat Budget in Arctic at now>.**



Input heat	Output heat	Debt heat
air＝84		atmos warm＝0.
ocean＝6		ocean warm＝4.5
solar＝30	CR＝115	Ice decay＝0.5W?
sum＝120W	**sum＝115W**	**＋5W/m².**

[１]：Heat Budget in Arctic:



You could see that **debt increase**(red box) is stronger enough(5～120W),while that of **repayment**(blue box) is far weaker(0～41W). The more accurate details could be in later. **The bankruptcy** due to ice lid vanishing is evident even only by this figure !!.Above reality at now(2014) some are **uncertain**,but it is a consistent budget model.The data are

The large-scale energy budget of the Arctic(2007)

https://courses.eas.ualberta.ca/eas570/arctic_energy_budget.pdf

http://www.colorado.edu/geography/class_homepages/geog_4271_f12/lectures/notes_2.pdf

The risky aspect of Arctic Bank is quite similar with global financial(Banks) crisis,both are due to **debt the imbalance of budget**..Note bankruptcy of **Arctic Bank** would cause our extinction(**Arctic methane catastrophe** by debt heat in ocean),while the other banks are not.**So long as ice lid vanishing rate is far stronger than the radiation increasing**.,we could not evade our extinction.That is,something **man made cooling technology** becomes decisive.Those who could save Arctic bank is nothing, but the other global banks.

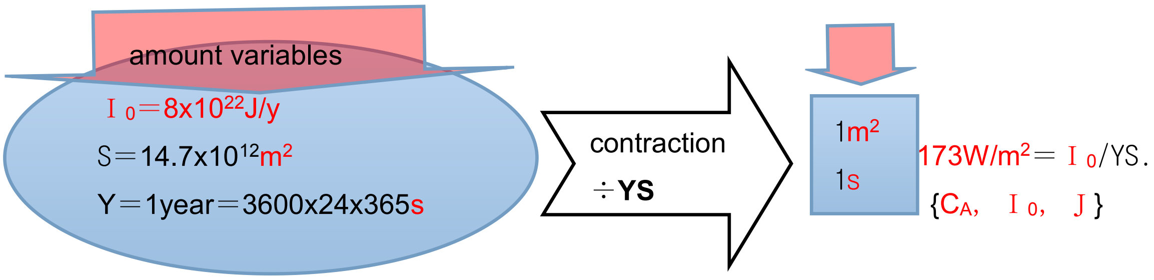
[２]： A Mathematical Model predicts the bankruptcy by a simple calculation.

Summary to tell,toward the catastrophe,rapid ice lid vanishing time rate＝da/dt would cause rapid temperature increasing,while cooling radiation increasing is far from cooling enough. We could not be expert in Arctic climate,but **judges** by this simple model.

Even though **data are rather uncertain**,the conclusion could be **decisive** !!!.

(0)：**the preliminary(measurement standard convention).**

(a)Input and output annual energy values(for example,solar input＝ I_0＝$8x10^{22}$J/y) in Arctic area(S=$14.7x10^{12}m^2$) is too huge,so those are transformed **$1m^2$x1second scale/year**.



(b)**Budget equation the interpretation**<this is a macroscopic averaged relation>.

As you see map of arctic ocean,it is very complicated environment,while our model Is to represent it only by **averaged few parameters**{t, T, a, @ ; C_A, I_0, J}.

t≡time in unit of years, T≡ocean averaged temperature in Kelvin unit,
a≡albedo≡averaged Arctic reflection(**cloud and ice surface**)rate for insolation input I_0.
@≡permeability of **cooling radiation**(CR)≡passing rate of CR into cosmic space

annual **heat debt increase** in Arctic ocean(of sea water temperature rise and ice decay)
＝heat capacity of the ocean(C_A)×ocean temperature rise/year(dT/dt)
＝**income**－**outgo**＝{solar＋(ocean air heat flow≡ J)}－{cooling radiation output＝@ σT^4}

C_A(dT/dt)＝{(1-a) I_0＋ J }－@ σT^4. <unit area and time with t＝years>
YSC_A(dT/dt)＝YS{(1-a) I_0＋ J }－@ σT^4}.<Total and annual with t＝years>

Especially note that time variable＝"t" is usually in **seconds** in dynamics of MKSA unit. However climate science's main concern is **years change**.Note intensity variables{t, T, a, @}are **no change** by **YS scaling transformation**.

(1)**Heat capacity of ARCTIC OCEAN**≡heat energy amount for 1 ℃ rise.

C_{AF}=**Arctic ocean total(long time)heat capacity;** $C_{AF\#}=C_{AF}/YS$=159W/m²K.

=14.7x10¹²m²×1225m×1020kg/m³×4.02x10³J=7.4x10²²J/K.

C_A=**Arctic ocean active(annual) heat capacity;** $C_{A\#}=C_A/YS$=79W/m²K.

=14.7x10¹²m²×600m×1020kg/m³×4.02x10³J/kg=3.7x10²²J/K.

C_A(dT/dt)=heat for temperature rise=(dT/dt) for heat capacity C=**debt heat rise**/year.

= **(heat input−heat output)**/year.

=**net {insolation+ocean atoms heat inflow((2))−cooling radiation from TOA((3))}.**

=(1-a) I_0+ J −@σT⁴≡ΔF=5W/m²(at now)=**<ocean warming+ice decay>**

＊dT/dt=ΔF/C=5W/159W～5W/79W=**0.03℃～0.06℃/y<Arctic ocean temperature rise !!>**
Now sea temperature rise=0.03℃/y seems agreement by observations !!!
→ $C_{AF\#}=C_{AF}/YS$=159W/m²K.
http://www.grida.no/graphicslib/detail/arctic-ocean-surface-temperatures_1242

(2)**insolation+ocean atoms heat inflow−Cooling Radiation(CR=LW)=debt heat rise.**

ΔF≡P+(J_1+J_2)−LW=(1-a_i)(1-a_c) I_0+ J −@σT⁴.=5W/m².(at Now),

(a) J =atmospheric heat inflow(J_1)+ocean(J_2)=90W/m².

this would increase as time goes on.

(b)insolation input≡P≡(1-a) I .

I_0=173W/m².→ I =(1-a_c) I_0=120W/m² → P≡(1-a_i) I ≡(1-a) I =(1-a_i)(1-a_c) I_0

net insolation input at now P=(1-a_i) I =30W/m². → .a_i=a=1−30/120=0.75.

(c)**Cooling Radiation(CR=LW=long wave) at Top of Atmosphere(TOA):**

LW≡@σT⁴=**CR passing probability**(@) to space x **black body radiation**(σT⁴)

There is nothing,but this **unique outgoing heat flow** that can cool deadly earth.

(3)**How much cool by 1℃ temperature rise ?. The actuality is a little !!!**

@σT⁴=115W/m.(at now). → (dT/dt)∂Po/∂T=(dT/dt)4@σT³=?

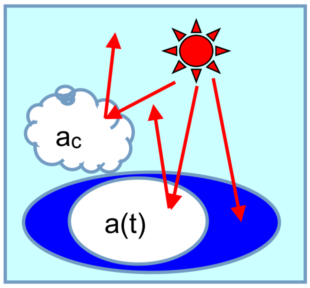
(4)**Annual average Arctic temperature estimation??** : σ=5.67x10⁻⁸W/K⁴m².

	effective temperature T=[Po/@σ]^(1/4)	CR sensitivity=4@σT³ for T
~~@=0.60~~	~~241K~~	1.90
~~@=0.55~~	~~246K~~	1.86
@=0.50	252K	1.81W/m²K.
@=0.45	259K	1.77W/m²K.
~~@=0.40~~	~~267K~~	1.73
~~@=0.35~~	~~276K~~	1.67

⑸**Average effective ice albedo change rate**＝$da(t)/dt$.

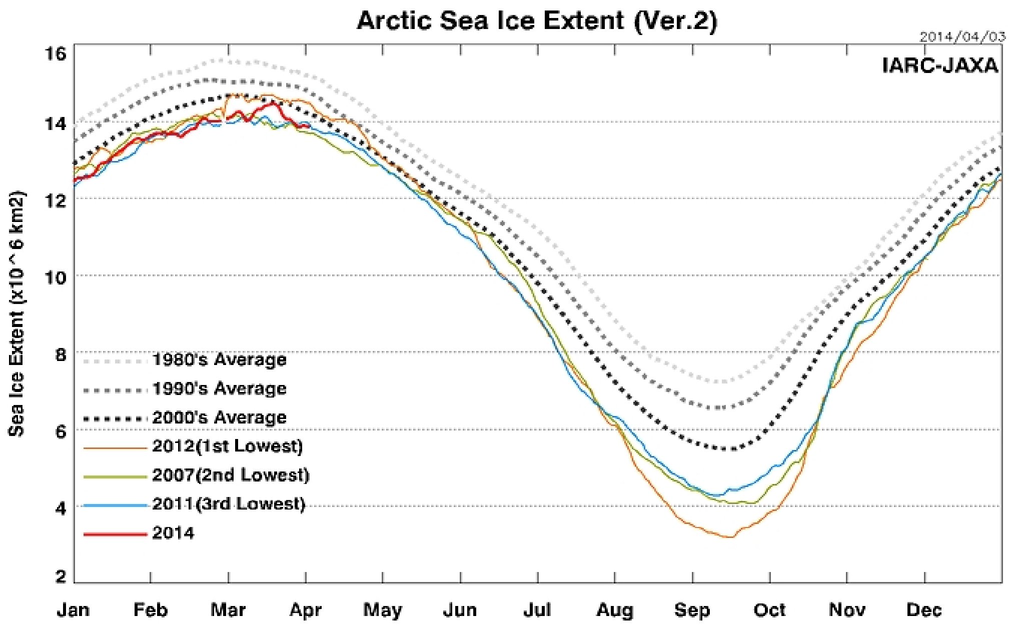
⒜insolation input≡$P(t)$≡$(1-a(t))\,I$ ≡$(1-a(t))(1-a_c)\,I_0$.

⒝$1 \geqq a(t)$＝[ice extent(t)/max ice extent=ocean extent]$\geqq 0$.

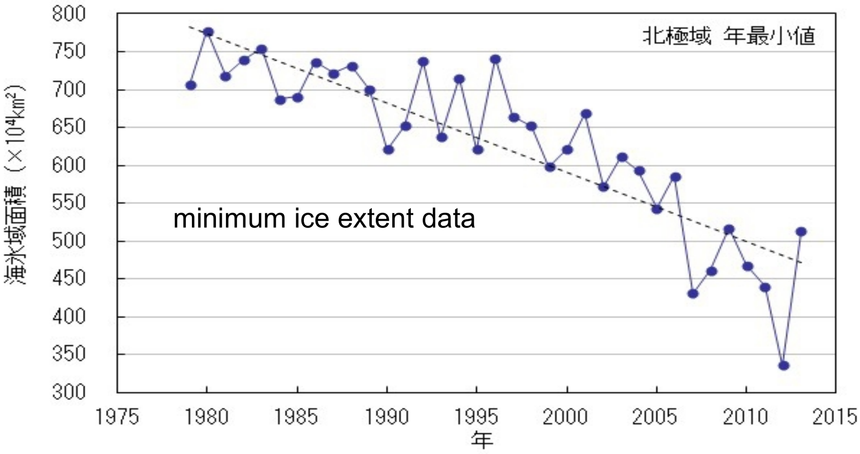


⒞**ice extent monitor**

http://www.ijis.iarc.uaf.edu/en/home/seaice_extent.htm



S_0＝$15\times10^{12}m^2$	max	min	center	albdo(area)＝center/S_0
1980av	15.5	7.5	11.5	0.77
1990av	15	7.0	11.0	0.73
2000av	14.5	5.75	10.1	0.67
2007	14.0	4.0	9.0	0.60
2011	14.0	4.2	9.1	0.61
2012	14.3	3.2	8.75	0.58



北極域の海氷域面積の年最小値の経年変化(1979年～2013年)
青色の折れ線は北極域年最小値の海氷域面積の経年変化を示す。点線は変化傾向。

http://www.data.kishou.go.jp/kaiyou/shindan/a_1/series_arctic/series_arctic.html

(d)$(da_i/dt)/a_{i0}$＝(800−450)/800(a_{i0})x40y≒−0.01/y.....**ice albedo decline rate**.

Note this value was employed in thie report.

(6)**Conclusion:insolation increasing is far stronger than that of cooling one.**

cooling radiation response＝−ΔT(4@σT^3) and **albedo change**＝−Δa I .

C(dT/dt)＝(1−a(t)) I ＋ J −@σ$T(t)^4$≡ΔF(t).

−(da/dt) I −(dT/dt) (4@σT^3)＝C(dT^2/dt^2)(dT/dt).

I ＝120、4@σT^3(@=0.5、T=252)≒1.8、(dT/dt)＝0.03～0.06℃/y at Arctic.

left term＝0.07x173−0.06x1.8

＝**1.2−0.11**≒1.2＝C(dT^2/dt^2)(dT/dt)＝C(dT^2/dt^2)x0.06℃/y＞0.

＝**1.2−0.06**≒1.2＝C(dT^2/dt^2)(dT/dt)＝C(dT^2/dt^2)x0.03℃/y＞0.

＊Temperature rise trend(＝(dT^2/dt^2))is positive,due to at least 10 times stronger albedo effect than cooling radiation rise., so sea temperature rise is more than linear rise !!.

[３]：Model Prediction by Arctic heat budget equations.

This report do not refer to seasonal change,but **smoothed years change**.The aim is years estimation of **full ice vanishing in years**,but not summer seasonal one.

⑴**The physical foundation:**

Heat budget for Arctic is simple to see,but essentially strict, and even becoming a dynamics. Because debt in budget is also **debt increasing rate** (per year) and is proportional to (input －output).that is .dD(t)/dt＝(I(t)－O(t)).This is nothing,but a dynamics equation.

Note the most of debt heat at now **5W/m²** is partitioning to **ocean warming**(90%)and to **ice decay**(10%?).This partitioning would turn to increase ocean warming with ice decay going on

⑵**ocean warming**(90% at now and the last stage is 100%)**:**

A heat has proper feature of **diffusional** into matter space due to **2nd low of thermodynamics**.Especially in ocean,the rather rapid diffusion is due to **sea water turbulence** in depth about 600m(**seasonal thermocline**).Of course,the last stage is full depth 1225m (average in Arctic ocean). In this report,sea water temperature rise seems 0.03℃/y so it become evident to empoy full depth heat capacity⑶.As the consequence ocean can be considered a heat capacity with a parameter C_A(1225m)＝7.4x10^{22}J/K. Or .$C_{A\#}$(1225m)＝**159W/m²K.**

⑶**Ocean temperature rise/year**(＝dT/dt) by ΔF＝**5W/m² heat input.**

C_A(dT/dt)＝ΔF.　→　dT/dt＝ΔF/C_A＝5W/m²/159W/m²K＝0.031℃/y、

This value may be told rather not good agreement with 3 times larger than average global temperature rise of 0.02～0.03℃/y or **more**. Note global debt heat(**radiative forcing**＝1.6W/m²) ,while that of Arctic＝5W/m²/1.6W/m² is 3 times larger.

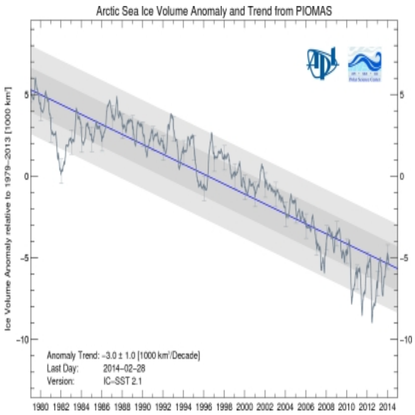
http://www.grida.no/graphicslib/detail/arctic-ocean-surface-temperatures_1242

⑷**Temperature singularity in co-being environment of ice and sea water.**

In salty sea water ,the temperature is told －**2**℃.That is **no temperature change** in those boundary of sea water with ice.However most of **99% sea water** may be apart from ice surface.It is evident that Ice melting heat is to be transferred from warmer water zone to such ice surface boundary zone by **sea currents**.So those sea water temperature change is massively possible in macro view.**By sea water temperature rise -2～0℃, Ice lid would be full vanished.**

(5)**heat partition rate for ice decay/y≡ε(t) in total ocean warming**＝C_A(dT/dt).

http://psc.apl.washington.edu/wordpress/research/projects/arctic-sea-ice-volume-anomaly/#



Heat for ice decay

(3.0x1000Km^3/10y=3.0x10^{12}m^3＝3.0x10^{11}m^3/),

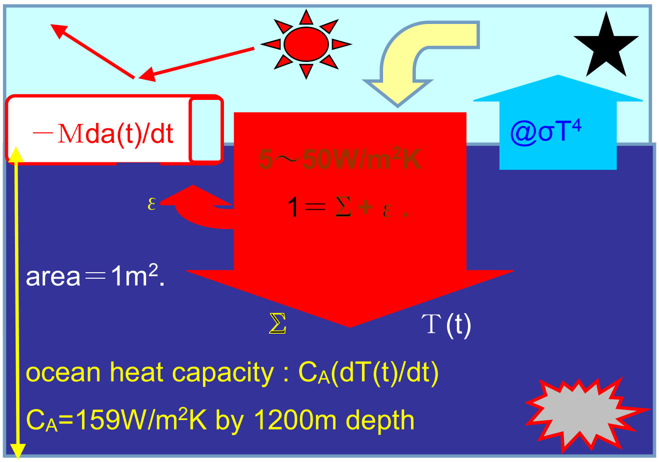
normaliztion factor SY≡area arctic ocean

X seconds in a year＝14.7x10^{12}m^{2}X3066x24x365.

(a)**heat for annual ice vanishing(≡Pice≡Pi)**

＝3.0x10^{11}m^3/yX917Kg/m^{3}x334.7KJ/Kg＝9.2x10^{19}J/y.

Pi＝9.2x10^{19}J/y/4.636x10^{20}m^{2}s＝**0.2W/m^2(~0. 4 W/m^2)**!



This is a kernel point of this approximation model theory.

(b)Debt heat＝C_A(dT/dt) must be **partitioned** both to ocean warming and ice melt by ratio

Σ(sea)＋ε(ice)＝1.

Then Σ is more than 90%,while ε is less than 10% ?.

*note:Ocean heat capacity is 159W,so once heat input is 50W,2 year temperature risecould be 2x50/159＝0.6℃,it would be dangerous for sea flor with Methane reservoir.

(c)As ice extent＝a(t) decreasing on,the melting heat amount/year seems to become less, while temperature gradient would be increased to cause more heat flow ??.

(d)**This is a coarse assumption that "ε(t)" is rather constant .**

－Mda(t)/dt＝εC_A(dT(t)/dt)...............(5) **albedo equation.**

(e)M is **ice melting heat coefficient** of (heat amount /unit area ,unit time).

M(t)＝1m^{2}x2m(~**ice thickness**)X917Kg/m^{3}x334.7KJ/Kg/3600x24x365s≒20W。

(f) ε≡(−da/dt)M/C_A(dT(t)/dt)＝(0.2～0.4W)/5W＝0.04～0.08.

(g)(−da/dt)M＝0.2～0.4W/m^2. <observed value>

→ {0.2～0.4W}/20W＝0.01～0.02＝da/dt 〈albedo **decreasing value** by observation〉

(6) Sea water temperature equations:

$C_A(dT/dt)=(1-a(t))I+J-@\sigma T(t)^4$. . ....(6) $\langle a(t)\equiv a_i(t)$、$(1-a_c)I_0\equiv I$, $J=$constant$\rangle$

$-Mda(t)/dt=\varepsilon\Delta F=\varepsilon C_A(dT(t)/dt)$..............(5) **albedo equation.**

(7) solving simultaneous equation.

Arctic ocean temperature equation. <d J /dt=0 is assumption at this time>

$d^2T/dt^2=<(\varepsilon(t)I/M(t))(dT/dt)-(4@\sigma/C)T(t)^3>(dT/dt)\equiv\tau^{-1}(dT/dt)$.

$C(d^2T/dt^2)=-(da(/dt)I-4@\sigma T(t)^3(dT/dt)$. $C(d^2T/dt^2)=(\varepsilon CI/M)(dT/dt)-4@\sigma T(t)^3(dT/dt)$.

$d^2T/dt^2=(\varepsilon I/M)(dT/dt)-(4@\sigma/C)T(t)^3(dT/dt)$.

(8) the solutions:

$d^2T/dt^2=\tau^{-1}(dT/dt)$. → $dT/dt=\tau^{-1}T+A$. → $A=dT/dt-T/.\tau$ →

$A\exp(-t/\tau)=(d/dt)<\exp(-t/\tau)T(t)>$. → $\exp(-t/\tau)T(t).=-(A\tau)\exp(-t/\tau)+B$ →

$T(t).=B.\exp(t/\tau)-A\tau$.

(a) **Temperature solution:**

$T(t).=\tau(dT(0)/dt).\exp(t/\tau)-\tau\, dT(0)/dt$.

$\langle T(0)\equiv 0$℃, $(dT(0)/dt)\equiv$observed value at now$=0.03\sim0.06$℃$\rangle$.

$-da(t)/dt=\varepsilon C_A(dT(t)/dt)$. → $a(t)=-\varepsilon C_A T(t)+a''$.

(b) **albedo solution:**

$a(t)\equiv<a(0)-\tau(da(0)/dt)>+\tau(da(0)/dt)*\exp(t/\tau)$.

$a(0)\equiv$observed value at now$\fallingdotseq0.75?\rangle$.

$da(0)/dt\equiv$observed value at now$\fallingdotseq-0.01\sim$or less$\rangle$.

(c) $(da/dt)/(dT/dt)=(da(0)/dt)*\exp(t/\tau)/(dT(0)/dt).\exp(t/\tau)=(da(0)/dt)/(dT(0)/dt)$

Note this value is time invariant only for this peculiar model.

(d) $-da(t)/dt=\varepsilon C_A(dT(t)/dt)$ → $-da(t)/dt=-(da(0)/dt)*\exp(t/\tau)=\varepsilon C_A(dT(0)/dt).\exp(t/\tau)$

→ $-(da(0)/dt)/(dT(0)/dt)=\varepsilon C_A=-(da(t)/dt)/(dT(t)/dt)=$constant in time..

⑼**0th approximation:**

$d^2T/dt^2 = < \varepsilon I/M - (4@\sigma/C)T(t)^3 > (dT/dt)$.

Now we approximate 2^{nd} term $\equiv -(4@\sigma/C)T(0)^3(dT/dt)$.

$4@\sigma T^3$(@=0.5、T=273)/C_A≒2.3/159=**0.015**≡ ξ ≡roughly constant,but very bit ?!!。

$\tau = (\varepsilon I/M - \xi)^{-1}$
＊Ｉ＝(1−a_C)=120W/m^2.
＊ ε ≡(−da/dt)M/C_A(dT(t)/dt)=(0.2～0.5W)/5W=0.04?～0.08.
＊M≡1m^2x2m(～**ice thickness**)X917Kg/m^3x334.7KJ/Kg/3600x24x365≒19.5～24.4～**20**
＊ ξ ～0.015

	τ =22y	τ =9. 5y	τ =6. 1y	τ =4. 4y	τ =3. 5y	τ =2. 9y	τ =2. 5y
Ｉ	120	120	120	120.	120.	120.	120.
ε	0.01	0.02	0.03	0.04	0.05	0.06	0.07
M	20	20	20	20	20	20	20
ξ	0.015	0.015	0.015	0.015	0.015	0.015	0.015

Summary:this report is due to ***accounting principle with partitioning concept***.

Ⅰ:**Sea water temperature equations:**〈a(t)≡a_i(t)、(1−a_C)I_0≡Ｉ〉

⑹C_A(dT/dt)=(1−a(t))Ｉ＋Ｊ−@σ$T(t)^4$. ....**heat budget equation.**

⑸−Mda(t)/dt=εC_A(dT(t)/dt).............**ice melt by partitioned heat of** C_A(dT/dt)

These are rather not a physics,but **accounting principles** the tautology ?!.

Ⅱ:**Time constant:**

$d^2T/dt^2 = < \varepsilon I/M - (4@\sigma/C_A)T(t)^3 > (dT/dt) \equiv \tau^{-1}(dT/dt)$. 〈 ξ ≡$(4@\sigma/C_A)T(0)^3$〉

The solution is exponential curve with time constant $\tau = (\varepsilon I/M - \xi)^{-1}$～**few years ?!!**

Ⅲ:**discussion:**

Strictly to tell, τ is not constant,but rather time dependent.However even as those are,

$\int_0^t du/\tau(u) = t/\langle\tau(0<u<t)\rangle$、<<theorem of middle value of integral>>

So ,something time constant could be in coarse average meaning.Therefore estimation on parameters{$\tau = (\varepsilon I/M - \xi)^{-1}$}could be not so much wrong.

⑽Albedo and Temeperature Solutions a possible case ?!.

(a)**Data for the graph.**

$C_A(dT/dt)=(1-a(t))I+J-@\sigma T(t)^4$.

$d^2T/dt^2=(dT/dt)< I\varepsilon/M-(4@\sigma/C_A)T(t)^3>(dT/dt)$

$-Mda(t)/dt=\varepsilon C_A(dT(t)/dt)$...

$\varepsilon\equiv(M/C_A)<-da/dt/(dT/dt)>$. ....original definition.

$\sim(0.2W\sim0.4W)/5W.=0.04\sim0.08$...............observed value.

$\tau\equiv< I\varepsilon/M-(4@\sigma/C_A)T(t)^3>\equiv(\varepsilon I/M-\xi)^{-1}=(<(da/dt/(dT/dt)>(I/C_A)-\xi)^{-1}$

$<da/dt/(dT/dt)>=<\varepsilon\Delta F/M>/<\Delta F/C_A>=<\varepsilon C_A/M>$

(b)**possible estimated values.**

$I=120,,C_A=159W/m^2K.$ $M=20,$ $\xi=0.022.$	$\tau=(<(da/dt/(dT/dt)>(I/C_A)-\xi)^{-1}$. da/dt=observed value . dT/dt=possibe value M/C_A=possible values, ξ=possible value				
$\tau=(\varepsilon I/M-\xi)^{-1}$	2.8y	4..2y	5.8y	7.4y	9.0y
$dT/dt=\Delta F/C_A=5/159$	0.02	0.03℃/y	0.04	0.05	0.06
$da/dt=\varepsilon\Delta F/M\sim-0.01$	-0.01	-0.01	-0.01	-0.01,	-0.01,
$I/C_A=120/159$	120/159	120/159	120/159	120/159	120/159
$\xi=4@\sigma T^3$	0.015	0.015	0.015	0.015	0.015
$\varepsilon=(M/C_A)<da/dt/(dT/dt)>$	0.063	0.042	0.031	0.025	0.021

(c)**Possibly reasonable DATA**{dT/dt,da/dt, τ} **for the graph:**

$< I=120W/m^2,,C_A=159W/m^2K.\ M=20,$

$*\ \tau=(<(da/dt/(dT/dt)>(I/C_A)-\xi)^{-1}=4.2y$

$*\ dT/dt=\Delta F/C_A=5W/m^2/159W/m^2K=0.031$℃/y.

http://www.grida.no/graphicslib/detail/arctic-ocean-surface-temperatures_1242

https://courses.eas.ualberta.ca/eas570/arctic_energy_budget.pdf ......$<\Delta F=5W/m^2>$

$*\ da/dt=-0.01/y.$

http://www.data.kishou.go.jp/kaiyou/shindan/a_1/series_arctic/series_arctic.html

$*\ I/C_A=120W/159W/m^2K.$

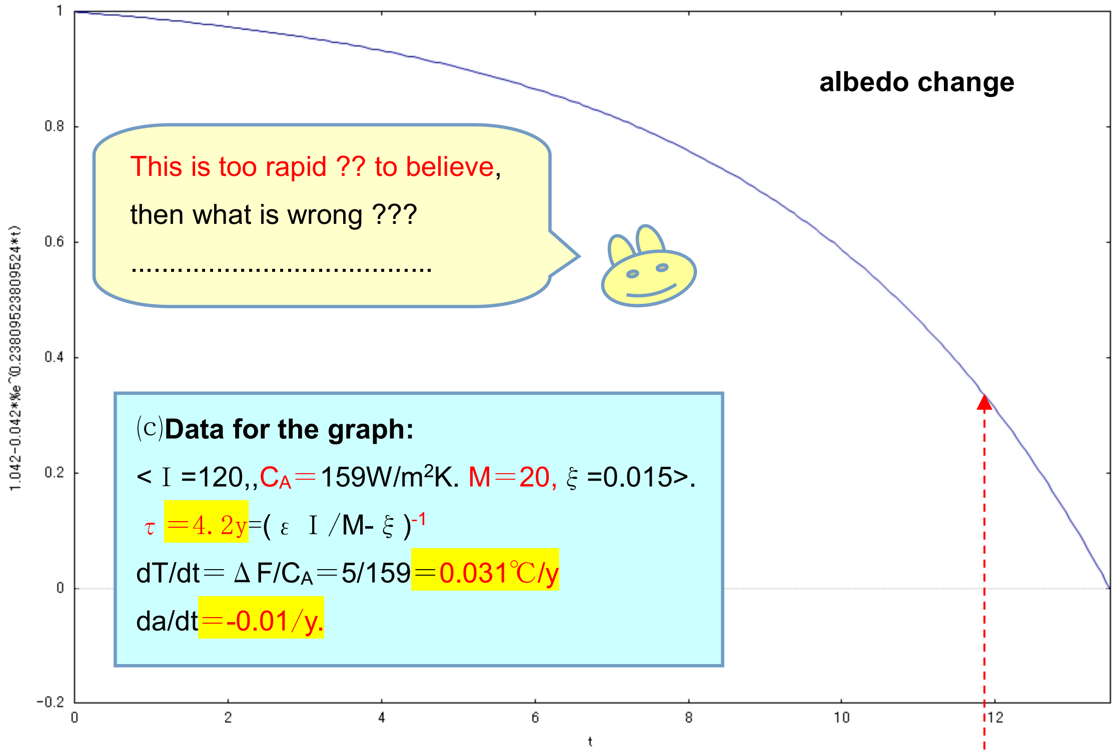
$\xi=4@\sigma T^3(@=0.5,T=273)/C_A\fallingdotseq2.3/159\sim0.015>$.This is very rough,but few value.

Following the graphs are faitufully due to above data and accounting principle equations.

You could claim for the details,however,fact that rapild ice vanishing may be decisive !!!.

(d)a(t)≡<a(0)＋ τ (da(0)/dt)>− τ (da(0)/dt)*exp(t/ τ).

```
plot2d(1+4.2*0.01-4.2*0.01*exp(t/4.2),[t,0,13.5]);
```



(e)T(t).＝ τ (dT(0) /dt).exp(t/ τ)− τ dT(0)/dt

```
plot2d(4.2*0.03*exp(t/4.2)-4.2*0.03,[t,0,13.5]);
```